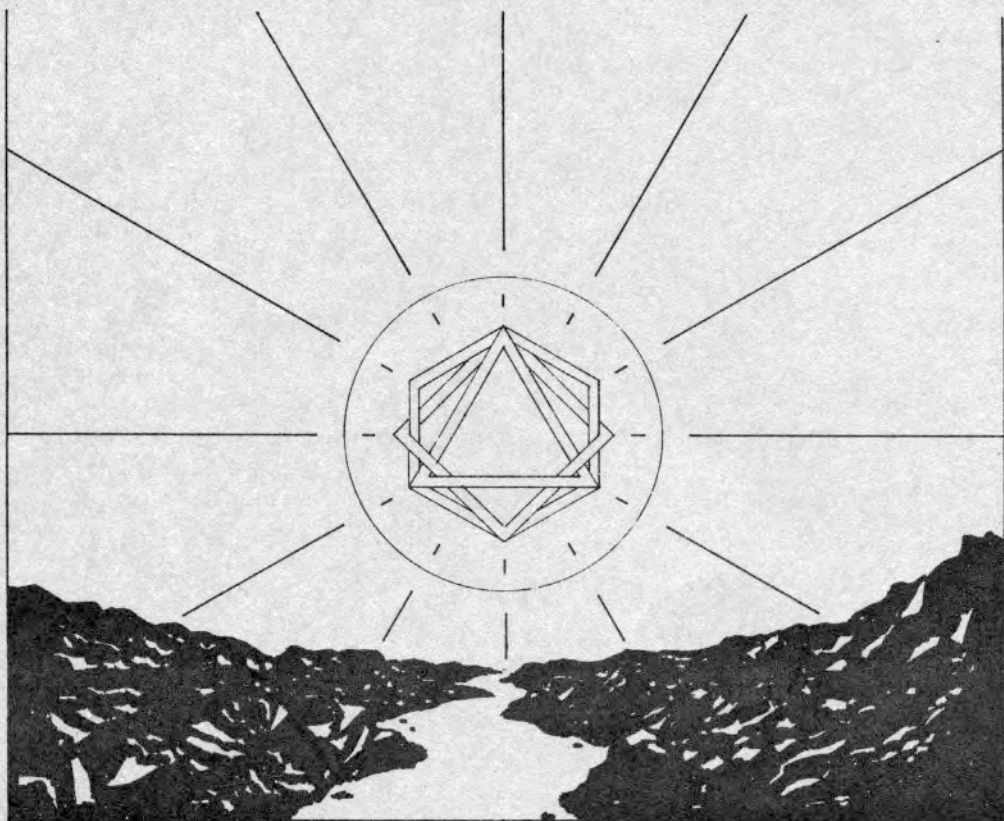


THE DOZENAL JOURNAL



NUMBER 4

SPRING

1986 / 1196

Published by:

The Dozenal Society of Great Britain
Denmead Hampshire England

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In association with the Dozenal Society of America
Long Island New York

ISSN 0260-4884

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It is with regret that we record here the death, last year, of our President, Sir Iain Moncrieffe of that Ilk, Bt, CVO, QC. It is many years since Sir Iain took an active part in the Society, but we remember his interest and patronage from the early days with gratitude.

D.H.

EDITORIAL

GALLONS...

THERE IS JUST ONE filling-station left in my neighbourhood where fuel is dispensed in gallons. It is not a case of struggling-on with worn-out pumps: the place is clean and up-to-date, with ICD pump registers. It simply happens that the business is privately-owned; and the owner takes the view that many motorists prefer gallons to litres. Quite a different story from the stations (most of them) owned by oil companies who seem to regard customers as captive consumers who will put up with anything. Long live private enterprise - and freedom of choice!

Somewhat sad is the fact that a lot of people now use the automatic pumps to buy fuel by the cost (£5 of fuel) and ignore the actual amount of fuel. They thus have no idea of their fuel-consumption rate, with the result that engine efficiency can fall without being noticed. People used to care about that sort of thing, and some still do; but metric innumeracy is spreading.

* * * * *

...VOLMS?

In this issue, Arthur Whillock reviews TGM. The aim of anyone who thinks seriously about measurement is, surely, to evolve a system which will satisfy not only laboratory work, but also that of the ordinary citizen who simply wants to do something practical. The decimal-metric system suits the former: laboratory work is essentially linear, much of it is abstract and it is conducted in an artificial environment. However, in the practical realm, to quote the American writer Stewart Brand (Journal 3), "metric fights your hand". The awkward, inflexible quality of the denary numoer-base is at the heart of the problem - as all dozenists know - which is why even the decimal-minded Romans avoided it for measuring. Furthermore, the SI units are unrelated in size to any human scale or dimension and so are difficult to estimate 'in the field', whereas our customary feet-and-inches are ideal for this purpose.

What are the properties of an optimal measuring system? We need a set of units based on sizes and parameters easily recognizable (and at least roughly checkable) by ordinary people, and precisely verifiable by science; we need arithmetical ratios and subdivisions which may accept the prime factor 5 when necessary, but must accept 2 and 3 (even in those scales where 5 is important, 2 and 3 are paramount); the system must, in those aspects of it which apply to everyday practice and commerce, be easy to learn and, dare one ask, interesting, so that it is a pleasure to use. (£sd was interesting and richly flexible; its bleak substitute, with too-large numbers is rightly described as "dismal"; ounces-and-pounds are simple and unassailably efficient in the kitchen.)

Readers may judge for themselves how well Tom Pendlebury's labours advance these ideals.

* * * * *

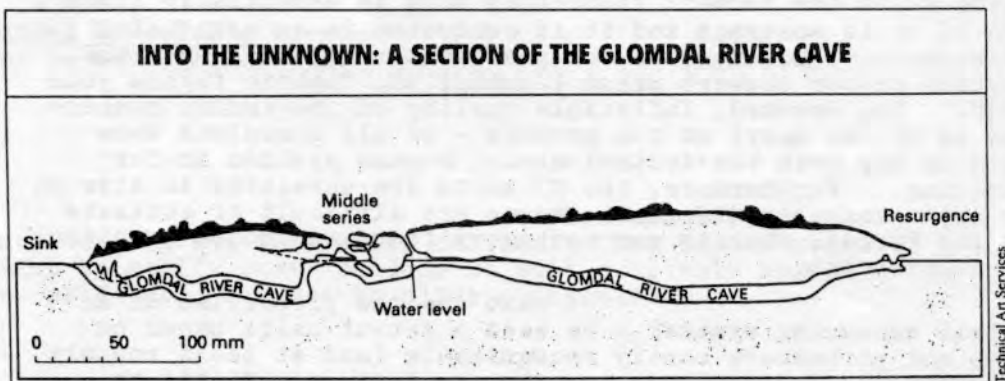
SETS...

Mr. Copley's meticulous analysis of digit-families will be food and drink to some readers; others may find it somewhat difficult. Mathematics at this level is an acquired taste, and I would not want anyone who has not acquired it to become discouraged thereby from contributing to our magazine. Remember that Einstein confessed himself to be baffled by the intricacies of income-tax!

While on the subject of contributions, I must say that I should like to hear from more of you. If you can write an article about some aspect of our concerns - and they are pretty widespread - then do. If you have a point to make or an opinion to express, then a letter is always very welcome, even if it is one of criticism. Do remember that the Dozenal Society's aims are towards genuine improvement in arithmetic and mensuration; towards better understanding of numbers and space and time by ordinary people; towards freedom of thought and the everyday commerce of life. We live in times when these things are being seriously eroded for political purposes; when mere changes - often for the worse - are hailed as 'moving with the times', with little heed paid to whether the said times are good or bad in prospect...

* * * * *

Mrs. G. Turner writes in this issue on the difficulties experienced by some children in forming the concept of number in abstract symbols. The illustration below shows the difficulty some adults have with the 'simple' metric system: the diagram would have passed through many hands on the way to being printed; yet it got through unchallenged!



22 TELEGRAPH SUNDAY MAGAZINE

* * * * *

Enclosed with this issue is a free leaflet explaining how decimals work in the kitchen. If anyone feels like making photocopies to leave around for the enlightenment of the public, then please feel free to do so!

* * * * *

LEST YE FORGET! A letter D on the address-label means that we should like a subscription, please. Best wishes for 1196.

The Editor.

NUMBERS.....Shaun Ferguson

"Simon Peter went up and drew the net to land full of great fishes, an hundred and fifty and three."

Simon Peter had gone fishing on Lake Tiberias, but at first he caught nothing. Then Jesus appeared on the lakeshore, and the disciple's luck changed miraculously. He at once caught 153 fish; and this makes it a double miracle, because 153 is an unusual number.

Firstly, it is a triangle number, and the sum of all numbers from 1 to 17. But, more importantly, it is the sum of the cubes of its own digits:

$$1^3 + 5^3 + 3^3 = 153$$

But 153 may have also appealed to the ancient mathematicians because it crops-up so often when we calculate with numbers that can be divided by three. Take 27, for example:

$$2^3 + 7^3 = 351 \quad (153 \text{ reversed})$$

Any other multiple of three will sooner or later give the same result; but you will have to repeat the process of cubing and adding. The number 123 needs 8 of these operations, while 99 needs three:

$$99: 9^3 + 9^3 = 1458; \quad 1^3 + 4^3 + 5^3 + 8^3 = 702; \quad 7^3 + 2^3 = 351.$$

Are there any other numbers like this? And, if there are in base ten, what about bases other than ten? Here is one which is true in base six:

$2^3 + 4^3 + 3^3 = 243$ (99 in base ten) - a multiple of 3 - so is it true only of multiples of 3? Try the number 2002, but put it into base twelve first.

See what you can discover.



"THAT'S RIGHT. HARDWOOD'S IN FEET AND SOFTWOOD'S IN METRES. THAT WAY, YOU KNOW WHICH KIND YOU'RE BUYING!"

The publication of TGM brings to fruition Tom Pendlebury's Herculean task of compiling a completely coherent dozenal system of metrology encompassing all aspects of science and technology, yet which is compatible with the measures, and their arrangements, appropriate to everyday experience and use. This is a conjunction singularly lacking in any proposal so far.

Measuring systems established in the past arose out of the real needs of their times and invariably started with a unit of length taken from familiar natural objects such as grains of corn or parts of the body. A length standard could then be reproduced by anyone, with sufficient accuracy for laying-out plots for cultivation or building. It would have been acceptable to all concerned but was quite arbitrary for any higher use.

When the time came to consider wider applications for measurement the real implications of the nascent technical era were scarcely realized. It was hoped to bypass chauvinistic variations of previous methods by adopting an Earth-based length "pour tous les peuples, pour tous les temps" but the result was still a misfit for scientific purposes by not allowing the value of any important constant to become a whole number. Naturally-evolved measures - arranged as they were in divisible groups - were at least well-suited to our understanding and physical needs. Those founded on the metre, with strict obedience to an inflexible numbering scale, failed both the scientific and social requirements of elegance together with comprehension and utility.

In TGM an initial unit of length is still required, but it is derived by insisting that the most important and widely-used physical relationship on this planet - the acceleration due to its gravitational attraction (g) - should be represented as simply as possible, i.e. by unity. For the person who is 'non-technical' there can thus be no numerical distinction between weight and mass, and the inclusion or omission of the constant, g , in dynamic calculations involving forces and the movement of masses becomes unimportant. Within a small unit of time - the Tim - being the hour divided by twelve to the

fourth power (just over a sixth of a second), the acceleration under an average value for g will be a little less than a foot per Tim per Tim. This length then becomes the fundamental unit for the system and is known appropriately as the Gravity Foot, or Grafut. With this, TGM has a practical and efficient spatial measure which, for good reason, appears in all national measures in one form or another.

Coincidentally, the Grafut is very close to the original Roman foot of $\frac{1}{2}$ '72 inches which in turn was based on sixteen Egyptian digits. Pendlebury modestly disclaims any credit for its discovery, saying that it comes to light when dealing with natural phenomena. Perhaps the Ancients were wiser than they knew!

With a length unit thus obtained, a volume is defined and its mass of pure water at maximum density becomes the unit of mass. The Maz is precisely one cubic Grafut, whereas a small error in constructing the initial decilitre cube means that metric weights were, and still are, not exactly related to volumes obtained from the length unit: the kilogramme is a little more massive than intended. Mechanical, physical, electro-magnetic, chemical and astronomical data are all re-structured into TGM with strict coherence maintained in dozens between units for scientific work. There is, however, good correspondence with familiar, practical measures - either directly or as auxiliary units - by virtue of the divisible arithmetic base which allows a branching-off at useful levels.

Doubling-up and halving-down are perhaps the commonest arithmetical operations we do and there are many ways of expressing them, from book sizes to musical notation. In TGM, logarithms to base two in dozenal notation provide a unique means for handling all ordinary ratios with a simplicity not found in other methods. Each chapter has interesting and sometimes refreshing comments on its subject-matter. Some will say that these are too simplistic, but it is difficult to decide on a level for wide appeal when presenting novel ideas. Students will be enlightened, especially as comparisons are made with Imperial and Metric methods. Those who know should not eschew different viewpoints which may help to transmit understanding to those less fortunate.

A minor disappointment is that is that the Grafut does

not quite coincide with the slightly longer metric foot of 30cm which, with its twelve 25mm "inches", is in common use in all but name. Even with points of contact it is not expected that TGM will be welcomed by science and industry to solve their problems, in view of the commitment to the present rival systems. It is offered by its author for the dozenist to use as appropriate and to demonstrate the value of a dozenal approach in all fields of enquiry. If the effort expended on developing an international measurement system had worked with more socially-acceptable material, we might now be in possession of an arrangement that would not only clarify scientific thought but forge links between all classes of use in lieu of the barrier that the decimal-metric method so often becomes; but, as Professor Aitken observed, "It is very early in the history of the world.....anything that is inefficient, even relatively so, will not last for ever."

* * * * *

.....DIZZIMALISATION!

So the pound note is to go, for the five-pound note is now only worth "ten bob" and the tanner a mere "quid".

One reason, I think, why people like the note is that it can't be confused with the "dolly mixture" coins we now have. In the old money the value of a coin in the pocket was judged mainly by its diameter, but the present-day Mint seems to be obsessed with making several of similar diameter, leaving us to sort them out by feeling for knurls, heptagonal corners and thickness (allowing for wear and tear, of course) - subtleties that elderly or work-worn fingers or those of owners who have had only one pint find not easy to distinguish.

What? No half-pee in decimal? Two-and-a-half is a natural division of ten, and four into fifty goes twelve and a half. But with the New Year's resolution four into ten will go three. Yet four threes are twelve, not ten! Once upon a time twelve pence made a shilling, and twenty shillings a pound. Now, twelve pee into a pound goes eight and a third!

Where is this "dizzimal" money floundering to?

Tom Pendlebury.

TWOANDSIXEQUALSONEEIGHTHOFAPOUNDANDTHREEANDFOUREQUALSONESIXTH

REGULAR POLYGONS have fascinated and influenced people throughout history. Few indeed do not respond in some degree to the symmetry and character exhibited by these shapes, each unique and yet plainly of a family.

Straight-sided, they all yet inhabit a circle precisely - in fact, the circle is their final expression, the Nirvana of regularity - and the rotational and lateral symmetries thus ordained give the regular polygons not only their visible perfections of mathematical form but also the indispensably useful geometry which some of them afford to Nature and engineering alike. The equilateral triangle, the square, the regular hexagon, the circle itself - all combine purity with practicality. Selected regular polygons are quite fundamental to our comprehension of space and are hence perhaps worthy of more appreciation and study than is presently fashionable in education.

It is hoped, in this article, to emphasize the way in which regular polygons can be seen as two-dimensional expressions of numbers: pictures, as it were, which illustrate more clearly than does arithmetic the connections between numbers themselves and what we can perceive, measure and use in the real world.

THE PLANE TRUTH

If what we see is to approach the actuality, it must be seen in two dimensions. While we may be impressed by the view of, say, the Parthenon when seen from an angle which shows it to be a solid and imposing structure, we are nevertheless obliged to move and look directly at one face at a time in order to discern the real shape: a viewpoint which gives the true angles and proportions of the front of the Parthenon renders its flanks invisible. We see but partially into the third dimension of space and optical illusions abound in line-of-sight perception (it is recorded that, during the building of Kilsby tunnel on the London and Birmingham railway, three men were killed as they tried to jump, one after the other, over the mouth of a shaft in a game of follow-my-leader)* and accurate perception is confined to the lateral plane.

*See "The Railway Navvies" by Terry Coleman (Pelican A903)

Hence, despite the availability of various methods of pictorial representation, engineers still insist on three-view orthographic projection for their working drawings and demand auxiliary projections to show the true shapes of angled surfaces. Architects know that the true plan determines the building. Plane, two-dimensional figures are thus paramount to our understanding of space and proportion; and the regular polygons - from line-segment to circle - constitute a definitive basic set by which order may be perceived in, and structure imposed on, our surroundings (Fig. i).

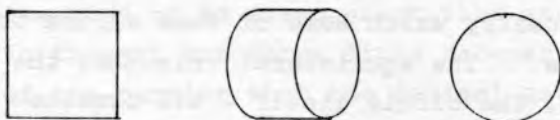


Fig. i

Not the least of these virtues is the insight given into the nature of numbers themselves: expressing numbers by presenting them as vertices of regular polygons shows very clearly properties which are not always evident from mere counting - particularly to children, but also to many adults who have never really understood numbers - and the educational need for such understanding will not, surely, be disputed in these days of declining numeracy?

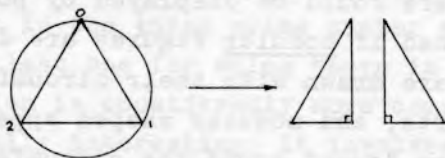
$$\begin{array}{l}
 1 \longrightarrow . \\
 2 \longrightarrow 1 + 1 \longrightarrow \cdot \cdot \longrightarrow \nearrow \\
 3 \longrightarrow 2 + 1 \longrightarrow \cdot \cdot \cdot \longrightarrow \cdot \cdot \longrightarrow \triangle
 \end{array}$$

One is a dimensionless point. Two is the first line number: we need two points between which to perceive length, the first dimension. It is possible to think of two as a regular two-sided polygon which has length, but no area.

The difference between two and three is not merely one unit: it is the difference between a line segment with only one dimension and an equilateral triangle which has two dimensions. Three is seen to be the first area or surface number and we could indeed measure area in terms of equilateral triangles. Each angle of the R-triangle is two-thirds of a right-angle and the shape will bisect into two drawing-board set-squares with angles of $\frac{1}{3}$, $\frac{2}{3}$ and 1 right-angle, each having a hypotenuse exactly twice the length of the shortest side. (It is important to note that these natural and convenient fractions of

the right-angle cannot be expressed by the Grade protractor, which is the basis of the decimal-metric system.)

Fig. ii



Constructing the regular trigon in its native circle (Fig.ii) - easily done by children - shows also that with three, 'handed' rotation is possible: we can travel round 0 - 1 - 2 - 0 or 0 - 2 - 1 - 0, whereas two is simply an alternating number (cf. electric motors). From these easy beginnings, it already emerges that three is a number of considerable significance (well before any considerations of trigonometry). The trigon shows the character of the number, in a way that arithmetic cannot, so that it can be understood by even the youngest. Evidently, any number-system that takes insufficient account of the importance of three is going to run into difficulties quite rapidly.*

FOUR is the square number, and the square is the optimum shape for area-measurement (we could tessellate equilateral triangles or regular hexagons to reckon area, but they have lengths in three directions, whereas the square has only the two directions). The properties of this right-angled figure, fundamental to mathematics and just about every practical activity that exists, are generally well-known and need little amplification here. It is worth mentioning, however, that four is also the first space number: four points will delineate a tetrahedron and so contain a volume.

$$4 \longrightarrow 2 + 2 \longrightarrow 2 \times 2 \longrightarrow \dots \longrightarrow \square \longrightarrow \diamond$$

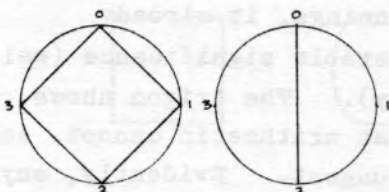
So, the square of the first prime number has its own properties far in excess of mere evenness and is a foundation-stone of any number structure; yet, ten will not accept four as a factor. Twelve, of course, will. A bisected square gives the companion set-square, with half-right-angles.

* 3 is the second prime number and, in conjunction with 2, controls the positions of all other primes: these are all members of the set $(2 \times 3)n \pm 1$, $n \neq 0$, which is the least set to contain them.

MODULAR FIGURES

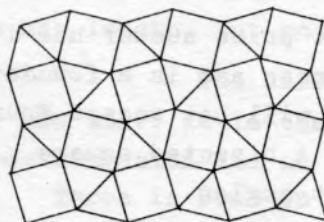
It was mentioned earlier that the characteristics of numbers could be displayed by polygons. This depiction is enhanced if modular figures are drawn: for each number N , circles are drawn with their circumferences divided into N equal parts; and modular shapes appear as, starting from a zero (top), we "jump" round the circumference by ones, then by twos, threes, and so on, joining successive points with straight lines until we arrive back at zero.

MOD. 4



Four thus gives a square (4/1) and a straight line-segment (4/2). The line-segment is, in this context, to be regarded as a two-sided figure and will appear with each even number.

With division of the circle into two, three and four equal parts, we have obtained regular shapes which are, above all, useful - indeed, one may say, indispensable - in that they form an indissoluble bond between mathematics and practical necessity. Line, triangle, square: length; the basis for geometry, trigonometry and engineering construction; the right-angle and optimum shape for areal mensuration. Not a bad score! Regular trigons and squares will tessellate in the plane with each other as well as with themselves -

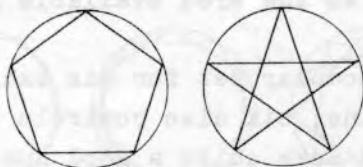


- and, so far, all seems harmonious and clear.

INTO THE SHADOWS

Five is a number at the edge of darkness. It is the third prime number and, for practical purposes, the last one for which there is an exact construction. The construction is considerably more complex than hitherto, but is mathematically interesting: it involves division of a line-segment in Mean and Extreme ratio - the Golden Section - and will divide the circle to give a regular pentagon*. The shapes in the set are more suited to decoration and psychology than to engineering and mensuration and are much favoured for symbolic and magical purposes.

MOD. 5



The powerfully-attractive star figure, used in Heraldry, is 'good' when this way up, and 'bad' when inverted.

Five is also the number of digits on the human hand, and it is a matter for regret that the primitive resort to finger-count has allowed this otherwise rather impractical number to usurp the rightful place of three as the second prime factor of the counting base in general use.

Regular pentagons will not tessellate in the plane with themselves or with other regular polygons (there is a three-dimensional relationship with a dozen: twelve regular pentagons form a regular dodecahedron), but, by comparison with the illumination given by two and three, five is a shaded and mysterious number: mathematically significant but of limited use. We should respect five and allow for its angular importance; but should not succumb to its esoteric charms when tackling arithmetic!

DAYLIGHT AGAIN

Six is the product of those all-important first two primes and gives us another useful shape: the regular hexagon. This is an optimum, and used as such by both Nature (honeycombs) and engineers (nuts and bolts). A plane tessellation of regular hexagons gives bees and wasps the most efficient possible accommodation for their pupæ; a

*See DOZENAL REVIEW No. *30

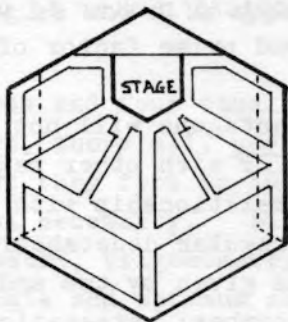
square nut is weak and an octagonal one slips in the wrench, but the hexagonal nut is just right.

MOD. 6



It is amusing to squeeze a pack of cigarettes and watch the hexagonal tessellation form itself as the area available gets reduced!

The modular set for six exhibits the equilateral triangle and the line; six also controls the location of prime numbers and could make quite a good number-base. The square is lacking, however, and so we must seek a little further for the best possible.



Chichester
Festival Theatre



The optimal properties of the hexagon are not lost on some architects: the Festival Theatre possesses the presentational virtues of Shakespeare's "Globe" combined with the strength and economy of steel-frame construction using ordinary straight stock, and is a much-admired, successful theatre.

SEVEN & ELF

The gloom we saw gathering about five becomes fully established with primes greater than five as far as circle-division is concerned. It is not possible to find an exact construction for seven and elf in the plane. The modular sets for these numbers show the pattern started with five: one convex polygon followed by a set of re-entrant star figures (dark stars?). A heptagonal curve-of-constant-width is used for coins by British decimalists: perhaps they

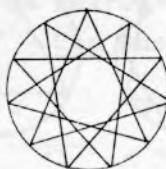
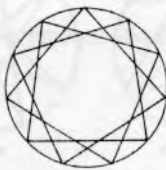
hoped thereby to make such coinage interesting to people being deprived of the richness and beauty of £sd?

MOD. 7



DECIMAL COIN

MOD. 2



The seven-day week arose from the Lunar cycle and accounts for the inclusion of seven as a factor in the larger English weights, which proceed in a binary fashion up to 4 Lb, but then go to 7 Lb, so that all the ensuing weights up to 1 ton are divisible by 7 as well as by 2. It makes sense if one is feeding animals or firing boilers, etc., and wishes to order fuel or food at a regular time in the working week, since the arithmetic is then easy*.

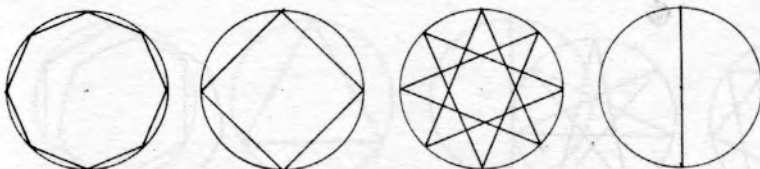
THREE CURATE'S EGGS

Eight, nine and ten are good in parts: the modular set for eight gives the square and a line-segment; nine gives the regular trigon; ten (decim) yields a regular pentagon, a line and two stars. Eight thus shows two useful shapes, but there is little to choose between nine and ten on this criterion: each have two re-entrant figures and one useful shape. Admittedly, two is a more important prime factor than three; and ten accepts two. The overall weakness of the decimal base is, however, shown very clearly by this two-dimensional analysis: running one's gaze along the modular set for ten shows at a glance the paucity of

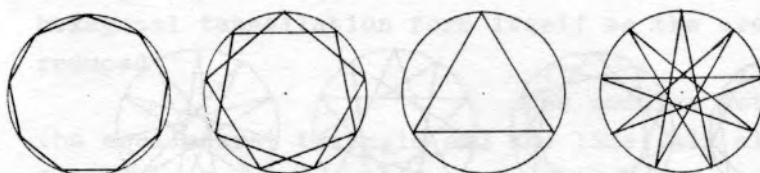
* The English ton is decimally 2240Lb, dozenally 1368Lb and is divisible by two, five and seven. 2 Lb per day = 1 st./week.

useful relationships afforded by this number.

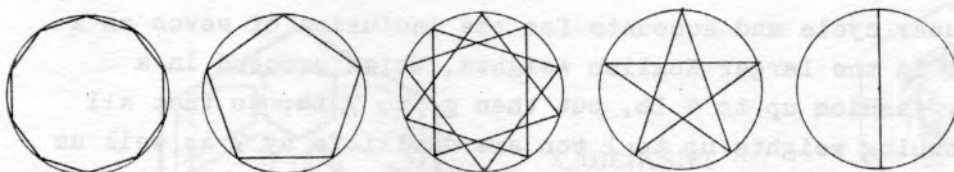
MOD. 8



MOD. 9



MOD. 2



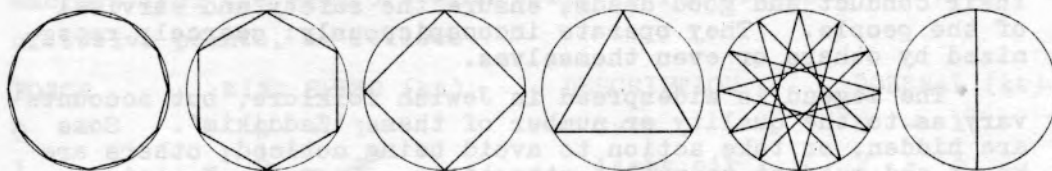
ARRIVAL

As we proceed, regular polygons with increasing numbers of sides grow more and more to resemble the circle - which is indeed a regular polygon with an infinite number of sides - and to lose their very angularity. It has, however, been well worth coming this far! Twelve is really a 'box of delights': the modular subjects of the regular dodecagon include those very shapes we know to be essential; and the dozen is the least number to do so.

The offspring of the dozen serve us well. Five of the six possible figures are convex polygons and four of these are essential to engineering and mathematics. If we omit the regular dodecagon itself (and it has significant properties, of which more in a later article), we get a ratio of four essential shapes to six possible. This two-thirds

ratio is exceeded by six, which gives unity in this respect: all the shapes for six are basic; but six will not provide the necessary square, and so twelve is the key number.

MOD. *10 (1 Dozen)



Need we search any further for a rational, serviceable number-base? Can there possibly be a better?

* * * * *

The table below compares the modular-plane-figure content of each number up to one dozen.

Circle divisions	No. of figures in the set	No. of convex figures	No. of essential shapes
2	1	1	1
3	1	1	1
4	2	2	2
5	2	1	0
6	3	3	3
7	3	1	0
8	4	3	2
9	4	2	1
2	5	3	1
2	5	1	0
10	6	5	4

The presence of a prime number is shown by a "1" in the third column.

It can be argued that, apart from the circle itself, there are only four essential regular polygons for practical purposes. Twelve has them all.

* * * * *

(To be continued)

There is an ancient tradition, mentioned in Islamic, Jewish and Persian writings, that the world is prevented from being submerged in its follies and wickedness by the presence in each generation of a small number of just men who, through their conduct and good deeds, ensure the safety and survival of the people. They operate inconspicuously: scarcely recognized by others or even themselves.

The legend is widespread in Jewish folklore, but accounts vary as to the quality or number of these 'Zaddikim'. Some are hidden, or take action to avoid being noticed; others are known and subject to public attention. Proverbs X praises the just man as the foundation of the world; early Talmudic writings give the number as thirty (from an interpretation of Genesis XVIII: 18 whereby God promised Abraham that the world would never lack so many such as he; and the Babylonian section refers to forty-five. The number was rounded-off to three dozen by a Babylonian teacher in the fourth century from a numerical interpretation in Isaiah XXX: 18 of the word for 'Him'.

It is thought, however, that the idea of thirty-six is not specifically Semitic, being taken from an Egyptian/Hellenistic notion of 36 celestial decans, each of which rules ten days of the year, and personifies them into watchmen of the Universe. Later, each sign of the Zodiac was given three faces and each decan bore the name of one of the Biblical characters from Adam to Ezra. This idea was eventually taken into Kabbalistic literature.

In Eastern Europe, for good reasons, Jewish people firmly believed that the thirty-six were always present to support them and confound their oppressors. They were given the name 'Lamedvavniks' after the numerals in the Hebrew counting system. Lamed, the twelfth letter in the alphabet, equivalent to our 'L', has the value thirty and Vav, six.

Whatever the details and origin of the legend - and this one follows the general pattern - it is a nice story with a ring of truth. The world is maintained by quite small numbers of ordinary people who, by their presence, with no special claim to merit or distinction, go about their affairs in times of upheaval to provide oases of sanity and continuity. Those who claim to be in control of events strut about in the limelight and manipulate the levers of power without considering that these are not necessarily connected to anything behind the panel, and any that are connected may do considerable damage before being reversed...

Could we modestly claim to be some of the three dozen?

References:

- The Messianic idea in Judaism. Gershom Scholem.
Allen & Unwin 1971
Encyclopedia Judaica p.1368
The Last of the Just (novel). Andre Schwarz-Bart
To Dusty Death (novel). Manes Sperbers.

WELL.....BLOW ME DOWN!

A salutary reminder (perhaps) that the power of decimalization should not be underestimated emerges from the Beaufort wind scale. The original scale is of twelve points, as follows.

FORCE	WIND SPEED (kt)	DESCRIPTION	DOZENAL (kt)
0	0 - 1	Calm	0 - 1
1	2 - 3	Light air	2 - 3
2	4 - 6	Light breeze	4 - 6
3	7 - 10	Gentle breeze	7 - 10
4	11 - 14	Moderate wind	11 - 14
5	15 - 18	Fresh wind	15 - 18
6	19 - 24	Strong wind	19 - 24
7	25 - 30	High wind	25 - 30
8	31 - 36	Fresh gale	31 - 36
9	37 - 42	Strong gale	37 - 42
10	43 - 48	Severe gale	43 - 48
11	49 - 56	Storm	49 - 56
*10	57 +	Hurricane	57 +

The decimalized scale has only ten points, having deleted 'High wind' and 'Hurricane'.

Sailors, and indeed all of us, must be grateful that the decimalists have abolished hurricanes; it must be a little unnerving, however, to have to jump straight from a strong wind to a gale!

* * * * *



"NO, Spriggs! However desirable it may be to relate decimal-metric units to human experience, we will NOT accept your suggested term for 'a unit puff of wind'".

A CANADIAN CLOCK?A letter from Paul Rapoport.

Dear Mr. Hammond,

As you may know from DSA Bulletin 51, I am interested in dozenal reckoning of time. For 15 years I have had a simple two-hand dozenal analogue clock running. Although I am interested in getting a dozenal watch built, building or altering watches remains difficult because of space limitations and standardization in the industry. But doing things with clocks is a little easier. Accordingly, a friend and I are building a dozenal digital clock. We wish to know whether anyone is interested in buying one, for then we can plan ahead better. A brief description of the clock follows.

The clock presents four digits, running from 0 to #. The leftmost digit changes once every one-dozeneth of a day (2 hours on the current clock), the next digit once every one-dozeneth of that (10 minutes), the next digit every one-dozeneth of that (50 seconds) and the rightmost digit once every one-dozeneth of that (4 1/6 seconds). For example, a time of 600.0 is noon, and 996.6 is 7:35:25 pm. There are 1000.0 units in a day.

The simplicity of this idea for an essentially dozenal metric clock has long appealed to me, and the prototype of the clock (now running) realizes what I predicted would work. Unfortunately, we cannot build a clock with four hands (an analogue clock), but fortunately, microchip technology makes a digital clock fairly easy to create. Other characteristics of the clock are these:

1. When completed, it will be about 7" x 11", attractively encased in a wood frame. Each of the four digits uses a 5-by-8 matrix of LEDs, which is approx. 2 5/16" by 1 3/8".
2. It runs on both 50-Hz and 60-Hz lines.
3. Since there is widespread variety in preference for numerals for the decimal ten and eleven, a switch in the back enables the user to choose from 8 pairs of such numerals stored in the clock's memory. These include both of the standard British numerals for ten, and both of the standard American pairs for ten and eleven.
4. Because the clock is a specialty item, very accurate and mostly hand-assembled, it will cost between £50 and £75. Even so, this is virtually the cost of parts plus labour.

* * *

Editor's note. The JOURNAL will be glad to print readers' comments on the above. Anyone wishing to write direct to Mr. Rapoport should send to: Dept. of Music, McMaster University, 1280 Main Street West, HAMILTON, Ontario.

AT BOILING POINT

Paul Jennings

I cannot prove this, but I just know that the two figures which even the least numerate people in the country could reel off automatically (not having actually learnt them at all, but having simply absorbed them somehow from the national air) are 1066 and 98.4. The Battle of Hastings (really the Battle of Battle - all right, then, Senlac, six miles from Hastings, since that is where they fought) and normal body temperature - in degrees Fahrenheit.

I bet not one in a hundred readers of this article knows what normal is in Centigrade - or "Celsius" - as they have recently taken to saying on television. In fact, I bet it does not even tell you in all the guff they give you when you take the car to the Continent - the Green Card, and all that stuff in eight languages about trouble with the car (extraordinary how "your clutch needs relining" sounds like one of the more mournful Schubert 'lieder' in German - 'Die Kupplung muss neu belegt werden' - while in Denmark, all meadows and cows and pigs and timberly farms and green copper spires, it is 'Koblingen traenger til at blive efterset', which sounds like some dreadful oath. "What's the trouble dear?" says your wife, as you return, black-faced, from the garage. "Ah, blive EFTERSET!" you reply, flinging a spanner on the ground).

Maybe it is because the thing is intimate and personal - dash it all, what could be more so than the question of whether one's own body is normal or feverish? - that Centigrade (or Celsius) is the very last bit of the compulsory EEC decimal system to have the slightest significance or reality for us of the good old pounds-shillings-and-pence generation. Half-a-crown, a guinea, a tanner, a bob, even a pound note - these were heavy with centuries of use behind them; real things, compared to which expressions like "fifty pee" will always seem an unreal abstraction.

Or take length. A metre, says the 'Dictionary of Scientific Units', "is defined in terms of the orange ($2p_{10}-5d_5$) line emitted by a krypton ($Kr86/30$) discharge in an Engelhard hot cathode discharge tube enclosed in

a cryostat at the triple point of nitrogen (63°K); one metre is represented by 1,650,763.73 vacuum wavelengths of this light"

Now they tell us. Those Jacobin madmen in their red nightcaps who cooked up this whole metric thing in 1793 did not know about krypton, any more than the few scientists whose heads had not been cut off and who mumbled something about the unit of length being " 10^{-7} of the earth's quadrant passing through Paris" and got this platinum bar kept in some vault which was THE metre - just because they decided to say so. Abstract, you see: whereas our yard comes from the Old English gierd or gyrd, a stick; and foot comes from, well, the length of a man's foot. What could be more physical and real than that?

Duodecimal, too. The 24-hour day, which we still have, goes back to the Babylonians and Sumerians, who saw the point of a number you can divide by 2, 3, 4 and 6 - not just 2 and 5. And even the French had twelve months in their new calendar (which Carlyle, of all people, translated so wittily: Vendemiaire, Vintagearious; Brumaire, Fogarious; Frimaire, Frostarious; Nivose, Snowous; Pluviose, Rainous; Ventose, Windous; Germinal, Buddal; Floreale, Floweral; Prairial, Meadowal; Messidor, Reapidor; Thermidor, Heatidor; and Fructidor, Fruitidor).

Before Dismalization we were always being told that we should be left behind in the international technological race if we did not adopt it. But the Americans have put men on the Moon and can hit a tin can or a missile with a three-inch nail from 3,000 miles up in space, and they still have gallons, miles, pounds and no intention of changing.

But none of these is so sadly missed as Fahrenheit. That in itself proves there is nothing insular about us, for it is clearly German, yet with an air of universality about it.

It does not matter so much that we do not think of that little bottle of beer as 275 millilitres, but still as some kind of half-pint; or that we always have to check whether it is about $2\frac{1}{2}$ lb to the kilo or $2\frac{1}{2}$ kilos

to the 1b; but it does matter if the weather man on television does not add the Fahrenheit equivalent in a kind of spoken brackets.

The other temperature scale, Réaumur, with its great clumsy degrees (only 80 from freezing to boiling, compared with Fahrenheit's 180) can be dismissed altogether. It does not sound like a man; more like a small town in northeast France, with a Second Division football team (Limoges 3, Réaumur 1). But Fahrenheit, the man, I always put with all those other German chemists brilliantly synthesizing indigo and such things in the 19th century, having names like Pfunzl, Hoffmeyer, Bayer or Stumpff. Celsius sounded earlier: one foot in medieval alchemy, the other in the Renaissance.

It turns out to be the other way round. Anders Celsius, 1701-44, was Swedish. Gabriel (Gabriel!) Fahrenheit, 1686-1736, was German and a fellow of the Royal Society. Well, rather Celsius than Centigrade (even if they are the same); and rather Fahrenheit than either of them. Bring him back!

* * * * *

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Editor's note: readers interested in the history and possible development of temperature scales are referred to DOZENAL REVIEW No.*32 or DOZENAL JOURNAL No. 1.

* * * * *

FAHRENHEIT'S ZERO

The standard temperature setting for the interior of domestic freezers is described as -18°C . At this temperature, bacterial activity is inhibited: life is at a standstill. It is thus the critical temperature for life at cellular level, and is (work it out!) at zero Fahrenheit: 0°F . Perhaps your friendly, local scientist should be told this fact of life!

EDUCATIONAL COMMENTSSTEALING THE GOLD?

The following is a quotation from 'Basic Draughting Practice', an engineering-drawing manual for school use from Nelson & Sons, written by R. Thomson of the Royal Military College of Science.

"The basic sheet (A0) is a rectangle of one square metre in area, the sides of which are in the ratio $\sqrt{2} : 1$. This shape, the 'golden section', in which the diagonal of a square becomes the long side of a rectangle based upon the square, has long been recognized as a perfectly balanced rectangle."

Good grief! It is nothing of the sort! The Golden Section results from the division of a line-segment in mean-and-extreme ratio, whereby the ratio of the shorter part to the longer equals that of the longer part to the whole line. Mathematically, for a line-segment divided into two parts, a and b,

$$a : b = b : (a + b)$$

and a rectangle in these proportions is held to be the most aesthetically pleasing possibility. The ratio occurs frequently in the proportions of living organisms and, interestingly, is approached more and more closely by successive pairs of numbers in the Fibonacci series:

0, 1, 1, 2, 3, 5, 8, ...

The approximate value of the Golden ratio is, decimally, 0.618... or dozenally 0'74ġ..., whereas the $1 : \sqrt{2}$ ratio of 'A' paper sizes is decimally 0.707.. or dozenally 0'859... , and it is noteworthy that our dear old foolscap paper size - eight by thirteen - is a close approach to the Golden Section and that 'A' sizes are not.

Just as the crazed, envious decimalists are trying to appropriate the ream, pretending that it means five hundred instead of forty dozen, so now they attempt to purloin the Golden Section for the greater glory of the metric sytem, even if it means lying to children...

* * * * *

THE TRUTH WILL OUT!

In the course of a recent schools' TV programme - 'Mathematical Investigations' - it was desired to show that the binary sequence: 1, 2, 4, 8, ... could, by grouping these numbers, give all the intermediate integral values, e.g. $1 + 2 + 4 = 7$, etc. For a practical example, the programmers had to use the traditional, elegant system of weights: 1oz, 2oz, 4oz, ... "Don't take any notice of ounces" said the presenter, hurriedly.

Well, of course! It would never do to reveal that the metric weight-sets need five weights, with two the same, to do the job of four in the oz/lb system, would it?

* * * * *

The NUMERAL is not the NUMBER !

Gwenda Turner

Our present numeral system relies on certain mathematical tricks which, unless explained, leave arithmetic in the realm of magic to most children (and hence adults). Numeral Codes were designed in an attempt to explain how a particular trick worked, that is, how ONE could be MANY.

If you distribute sweets unequally among children who are too young to have mastered counting, some will cry. Why is this? It seems that children must be able to recognize number before they can count. Numeral Codes make use of this visual recognition of number.

If a child can do the following sum:

$$I + II = III$$

why can it not immediately solve:

$$1 + 2 = 3$$

when it is the same sum? It is only visually different.

It is possible that the confusion arises because 2 is a single shape: up till now - in its own experience - when somebody gave the child two apples it saw @@ and when someone said "Take two biscuits", it was correct to take @@ but now, when someone else points out that one looks like 1 then (logically) the child knows that two ones will look like 11 - yet now 11 is wrong and, for no obvious reason, something shaped 2 is right! Numeral Codes attempt to bridge this gap.

Initially, children could be shown that two has been and can be written as II, but then shown that two ones can become one two by making the units form a single pattern:

$$\begin{array}{lll} // & \rightarrow & \angle \\ /// & \rightarrow & \triangle \\ //// & \rightarrow & \square \end{array}$$

Obviously, children can, and should be encouraged to, make their own patterns:

$$\begin{array}{lll} /// & \rightarrow & Y \\ /// & \rightarrow & \sqcup \end{array}$$

- and make up sums, using their own patterns for themselves and each other.

When children have experienced that many 'ones' can be represented by a single shape and how different shapes might be invented by different people to represent the same number, they should find it possible to accept that III can be written as 3 even if told that the teacher does not know why.

Now, if numerals based on the cardinal value of a number are easier to understand than ordinal numerals, it seems logical to extend the exercise and try to design a series of numerals in which the relationship is as obvious as in / and $\angle$. Modified Roman and other marks led to the shapes for the Decimal Code but it was the simplicity of designing a Code to base twelve that made the author investigate this base further and led to her conversion to dozens.

NUMERAL CODE FOR BASE TWELVE

The following patterns can also be made using matchsticks to reinforce the relationship to the number.

ONE TO SIX: / $\angle$ Δ λ $\bar{\lambda}$ $\bar{\lambda}$

SUMS: / + Δ $\rightarrow$ λ Δ + Δ $\rightarrow$ $\bar{\lambda}$

It is almost automatic.

Extending the pattern up to twelve -

SEVEN TO TWELVE: $\bar{\lambda}$ $\angle$ $\bar{\lambda}$ Δ $\bar{\lambda}$ λ $\bar{\lambda}$ $\bar{\lambda}$ $\bar{\lambda}$ $\bar{\lambda}$ $\bar{\lambda}$

- enables the teacher to demonstrate how another trick works, that is, how we can use a single stroke to represent more than 'one'. As, in all numbers above it, six is a component it should be possible to convince children that it is an unnecessary waste of time to draw the whole pattern when one of the diagonals will suffice. Hence, $\bar{\lambda}$ becomes / in numbers greater than six:

seven $\bar{\lambda}$ becomes 7 ; eight becomes 8 ;

nine $\angle$ $\bar{\lambda}$ becomes 9 .

If a child can accept this it might also find it easier to accept that when we come to place-value a dozen might also be represented by a single stroke; but that is another trick, beyond the scope of this article.

NOTE.

IF a dozen were the standard base it would be necessary to write the numerals at speed. For this a cursive form is more efficient and Δ would become 0. This means that another symbol, such as the Arabic $\blacksquare$ would have to be used for ZERO.

CURSIVE DOZENAL CODE:

1 2 3 4 5 6 7 8 9 10 11

NUMERICAL CONFUSION

The numerals for one to nine
Are taught by some as if 'sublime',
But the reason for their shape's not clearly there.
So, glassy-eyed, our puzzled children stare
While the one-to-one they knew so well
Gets lost forever in a Decimal hell.

* * * * *

SOME ADO ABOUT NOTHING

Mrs. Turner has also written-in with a suggestion that there should be a different symbol for zero with each counting base as a help in identifying the number scale, citing the square dot used in Arabic numerals to 'tick off' and empty column.

The zero or cypher we use derives from the Sanskrit word Sunya, used by Buddhists to denote a space which is empty but full of meaning - which is an apt description of its present function. To replace it with a different, involuted symbol would vitiate this concept and strain the existing stock of shapes available. We have enough difficulty in selecting two for the essential ones.

A further suggestion was to put a number or symbol inside the zero to make clear which base-number is used. This is better, but printing difficulties are involved.

Yet, the ideas have merit. Different shapes of the zero, retaining the closed-space form, could be adopted for different bases. Thus, binary could have a narrow 10, making the rows of digits more compact, and ternary, appropriately, could be triangular: 1 Δ . Variations on this theme are limited but 10 could be octonary, 10 denary, 10 dozenal and 10 hexadecimal.

Such small differences between forms of nought would be possible only in printed work and other methods of identifying the base would still be needed. It would, however, avoid the common fault of writing 10 for the base number and leaving the reader to infer which number is meant!

A.F.W

Although many may find it hard to credit, the latter phrase above is actually substituted for the former in the 'GOOD NEWS' Bible - one of the dreadful travesties of the Good Book now in very widespread use in our schools. Apart from the crass destruction of some of the most beautiful English ever written - destruction often losing the meaning as well as debasing the language - the literary (?) philistines responsible for this rubbish have also brought the benefits of decimals and metrication to the ancient writings.

I. Kings VII (or 1 Kings 7, as GN has it). Verse 23.

"Hiram (meaning Hiram) made a round tank of bronze, 2.2 metres deep, 4.4 metres in diameter and 13.2 metres in circumference." Verse 26: "The sides of the tank were 75 millimetres thick... The tank held about forty thousand litres."

Genesis VII, verse 15 (instructions for building the Ark):

"Make it 133 metres long, 22 metres wide and 13 metres high. Make a roof for the boat and leave a space of 44 centimetres between the roof and the sides." (No longer even called an ark!)

GOLIATH was "nearly three metres tall and wore bronze armour that weighed about fifty-seven kilogrammes..."

Making an altar (Exodus XXVII)

Originally: "And thou shalt make an altar of shittim wood, five cubits long, and five cubits broad; the altar shall be foursquare: and the height thereof shall be three cubits."

Now: "Make an altar of acacia wood. It is to be square, 2.2 metres long and 2.2 metres wide, and it is to be 1.3 metres high."

It was once said that the first few verses of St. John's Gospel were so perfect that they should be written in letters of gold; and so one might have hoped them to remain inviolate. But no: the poetry has gone, together with the meaning.....

Truly, this is Big Brother's Bible! Who's next? Shakespeare?

JUST TO SAY.....

Congratulations upon your letter published in today's DAILY MAIL.

My chief concern is to preserve the Statute measures for the future, at least in so far as possible, just as in a similar manner some countries are bilingual. And if there's to be any future in the development of mathematics then a duodecimal system has obvious advantages. In the opinion of myself and others, where the metric system fails so abysmally is in its total divorce from human measures. This was deliberate on the part of the Jacobins and the Club des Cordeliers and others. It was a tragic misunderstanding, the consequences of which are far-reaching. It does not matter overmuch on what scientific data a unit of measurement is based (providing, of course, that it be a satisfactorily accurate one) but it must enable the average man to envisage his connections to the Earth and if possible further.

As you are aware, this is the genius of British Imperial measures and follows a very ancient tradition.

Patrick Graucob
LONDON SW1

I was very interested in and thoroughly agree with your letter in the D. Mail and have always felt that there is a Society for everything in the UK (assuming yours is genuine!)

I was in groceries all my life; once one knew one's multiplication tables there was nothing easier than our Imperial units for mental calculation, e.g. $4\frac{1}{2}$ each = $4/6$ dozen = 54/- gross, since everything was packed in dozens (and still is). There was also 20/- cwt. = £20 ton. Gems like 99½ guineas added some variety!

S.J. Baker
SOUTH CROYDON

Six of one . . .

DERMOT PURGAVIE'S report on metrication in the U.S.—'Old ways linger on'—was somewhat mistitled. The decimal-metric system is an 'old way': it was invented by French academics in the 1790s and has been proving its mathematical and practical incompetence ever since.

At the time, Britain sensibly rejected it and went on to perform the Industrial Revolution—to the dismay and envy of the French!

Now we are becoming as daft as they are. God bless America, I say—our transatlantic friends may yet save themselves and us from the decimal dark age.

DONALD HAMMOND,
The Dozenal Society of Great
Britain, Denmead, Hampshire.

The decimal system goes in twos and fives. Everything else goes in twos and threes.

Norman Shaw
BEDFORD

MATHEMATICAL SECTIONDOZENAL DIGIT-FAMILIES & DIVISION-ALGORITHMS

G.N. Copley

Introduction.

A division-algorithm (DA) is a finite set of linear algebraic division-equations (DE), every member of which set can generate a unique dozenal digit, by taking the form:

$$y = Bx + r \quad (1)$$

In equation (1), y , B , $x \neq 0$ and r are elements of ring Z , the set of integers; y is the dividend and must not be a dozenal digit; B is the base and may be dozen > 0 or minus dozen (bardozen) < 0 ; $x \neq 0$ is the quotient and r is the remainder, or dozenal digit. For, given B and y , it can be proved that x and r , satisfying equation (1) and equations (2) and (3), given below, are unique integers. There are a dozen families (ordered sets) of dozenal digits $D_{|B|}^J$, where $|B|$, the absolute value or modulus of B , is defined as follows:

$$\begin{aligned} |B| &= B \text{ if } B \geq 0 \\ &= \bar{B} \text{ if } B < 0 \end{aligned}$$

Remainder r must satisfy equations (2) and (3):

$$|B| - (J + 1) \geq r \geq J \quad (2)$$

$$|B| - 1 \geq J \geq 0 \quad (3)$$

Dozenal digit-families

These are the dozen horizontal rows of Table 1. J is the rightmost digit in each row.

0	1	2	3	4	5	6	7	8	9	$\geq$	$\leq$	$\nearrow J$
$\bar{1}$	0	1	2	3	4	5	6	7	8	9	$\geq$	
$\bar{2}$	$\bar{1}$	0	1	2	3	4	5	6	7	8	9	
$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	4	5	6	7	8	
$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	4	5	6	7	
$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	4	5	6	} least-modulus digit-families.
$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	4	5	
$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	4	
$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	3	
$\bar{9}$	$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	2	
$\bar{\infty}$	$\bar{9}$	$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	1	
$\bar{\infty}$	$\bar{\infty}$	$\bar{9}$	$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	0	

The first row is D_{doz}^{∞} , the familiar dozenal digit-family: it has no negative digits (bardigits). The final row is D_{doz}^0 , which has no positive digits. These two families are unable to

represent negative integers when $J = \bar{\epsilon}$ or positive integers when $J = 0$, unless the base be bardozen. The remaining ten families ($\bar{\epsilon} \geq J \geq 1$) can represent, uniquely, all the elements of ring Z .

The DE modulus-condition. It can be proved of equations (1), (2) and (3) that $|y| \geq |x|$. It can also be proved that for $|y| = |x|$ the only DEs possible are, for the values of J appended to them:

$$1 = \text{doz} \cdot 1 + \bar{\epsilon}, \quad (J = 0) \quad (4)$$

$$\bar{1} = \text{doz} \cdot \bar{1} + \epsilon, \quad (J = \bar{\epsilon}) \quad (5)$$

$$1 = \text{bardoz} \cdot \bar{1} + \epsilon, \quad (J = 0) \quad (6)$$

$$\bar{1} = \text{bardoz} \cdot 1 + \epsilon, \quad (J = \bar{\epsilon}) \quad (7)$$

The dividends of these equations cannot be digits; it is easily verified that this is so for all four equations. There is also the question of whether the quotients are digits. If they are, then they cannot be followed by another DE with that quotient as dividend (see next section); so (6) and (7) are terminal DEs. If they are not digits, they can and must be followed by another DE. So, (4) and (5) are followed by another DE with their quotients as dividends. However, these DE that follow (4) and (5) are duplicates of (4) and (5), yielding the interesting conclusion that the division algorithm in such cases cannot terminate, but simply recurs to infinity.

The division algorithm

The foregoing elimination of $|y| = |x|$ from $|y| \geq |x|$ leaves $|y| > |x|$ to be considered, and gives the important result that if the successor of (1), the DE (1.1) obtained when x (not a digit) is used as a dividend, i.e.:

$$x = Bx_1 + r_1 \quad (1.1)$$

$$\text{with successor} \quad x_1 = Bx_2 + r_2 \quad (1.2)$$

and so on, employing the same remainder condition (2) throughout, then:

$$|y| > |x| > |x_1| > |x_2| > \dots > 0 \quad (8)$$

and, since a decreasing sequence of positive integers must terminate, then this terminal, a dividend, must belong to a terminal DE:

$$x_t = Bx_{t+1} + r_{t+1}, \quad (9)$$

in which x_{t+1} is a non-zero digit, r_{t+1} is a digit and x_t is not a digit. Hence, x_{t+1} can take $|B| - 1$ values, r_{t+1} can

take $|B|$ values and x_t can take $|B|(|B| - 1) = \zeta 0$ values. There is a greatest and a least signed terminal dividend which is not a digit.

As has been said, and can be proved, each DE generates a unique digit, its remainder, and so it follows that a DA generates a unique ordered set of digits, terminating in a DE which generates two digits. The ordered, juxtapositional set of $(t + 2)$ digits:

$$x_{t+1}, r_{t+1}, r_t, \dots, r_2, r_1, r,$$

conventionally represents, in one and only one way, the initial dividend $y \in Z$. It can be shewn, from the corresponding, ordered set of $(t+1)$ DEs forming its DA that:

$$y = (((\dots(x_{t+1})B + r_t)B + \dots r_2)B + r_1)B + r, \quad (\zeta)$$

in $(t+1)$ nested parenthetical form, and

$$y = x_{t+1}B^{t+1} + r_tB^t + \dots + r_2B^2 + r_1B + r, \quad (\xi)$$

in polynomial form.

Summary and comments.

1. Any non-negative integer can be represented, dozenally, in 1ξ digit-families, but not in D_{doz}^0 , which is a family having no positive digits (see Ex.3).
2. Any non-positive integer can be represented, dozenally, in 1ξ digit-families, but not in D_{doz}^ξ , which is a family having no negative digits.
3. For a given integer, its representations may be the same in different families because the intersections of the families share the digits of the given integer (see Table 1).
4. The negative sign (minus or bar) is conventionally augmented to D_{doz}^ξ and D_{doz}^0 to give $D_{\text{doz}}^\xi -$ and $D_{\text{doz}}^0 \bar{1}$ to enable them to represent all elements of Z . This use of the prefixed minus sign is equivalent, according to ring theory, to the multiplication of a following integer by minus one (-1) or bar one $(\bar{1})$, which is not a digit of D_{doz}^ξ , and, even though it is a digit of D_{doz}^0 , its usage still involves additions to these digital numeration systems.
5. Dozenal digit-families D_{doz}^J ($\xi > J > 0$) can be used to represent all integers.

6. Dozenal digit-families D_{bardoz}^J ($\xi \geq J \geq 0$) can be used to represent all integers. In particular, D_{bardoz}^J can express all numbers without using prefixed negative signs or bardigits (Note that D_{doz}^J and D_{bardoz}^J represent the same digit-family but it is sometimes necessary, for the sake of clarity, to use $D_{|\text{doz}|}^J$ to emphasize this).

7. Digit numeration systems have here been presented as dozenals, but the results apply to all bases. Those presently in use are the positive integers 2, 8, 2, *10 and *14, all of even parity.

Examples

1. On using the DA to convert positive integer 123456789220 (D_{doz}^ξ) to its D_{doz}^0 equivalent, it is found terminally to recur and is therefore not uniquely representable; endless representations are possible:

$$1\xi\xi9876543210$$

$$1\xi\xi\xi9876543210$$

$$1\xi\xi\xi\xi9876543210$$

... ..

This recalls recurring dozenal fractions of period 1, and may be given the form:

$$1\xi\xi9876543210,$$

representing a recurring integer of period 1.

2. The DA for the conventional dozenal-working conversion of dozenal 1195 to its bardozenal equivalent is:

$$1195 = -10 \cdot -119 + 5$$

$$-119 = -10 \cdot 12 + 3$$

$$12 = -10 \cdot -1 + \xi$$

Note that sequence (8) is exemplified by: $1195 > |-119| > 12 > |-1| > 0$; that the terminal of the DA happens to be equation (7); that the four equations of the DA ($t = 3$) give the five-digit bardozenal representation 1\xi235 of dozenal 1195, i.e. the representations, in accordance with (2) and (4):

$$1195_{\text{doz}} = (((1) \cdot -10 + \xi) \cdot -10 + 2) \cdot -10 + 3) \cdot -10 + 5,$$

and
$$1195_{\text{doz}} = 1(-10)^4 + \xi(-10)^3 + 2(-10)^2 + 3(-10) + 5.$$

The 20 possible terminals of DAs leading to the unique representations of integers in D_{doz}^ξ , with bardozen as base, can have greatest terminal dividend $1\xi = -10 \cdot 1 + \xi$, and least terminal dividend $20 = -10 \cdot \xi + 0$; all are negative bidigits (see Ex. 5).

Next is given the DA for the conversion of negative (minus-prefixed) dozenal -1195 to its bardozenal equivalent. The working is again in $(D_{\text{doz}}^{\xi}-)$:

$$-1195 = -10 \cdot 112 + 7$$

$$112 = -10 \cdot -11 + 2$$

$$-11 = -10 \cdot 2 + \xi$$

so that a DA of three DEs gives the four-digit bardozenal representation $2\xi 27$ of (augmented) dozenal -1195 . The terminal dividend in this case is -11 ($D_{\text{doz}}^{\xi}-$) and, from the terminal equation, it is 2ξ ($D_{\text{bardozenal}}^{\xi}$) to be found in row 2, dozenth column, of Table 2 now given.

TABLE 2: The $\xi 0 D_{\text{bardozenal}}^{\xi}$ terminal dividends.

ξ0	ξ1	ξ2	ξ3	ξ4	ξ5	ξ6	ξ7	ξ8	ξ9	ξ2	ξξ
20	21	22	23	24	25	26	27	28	29	22	2ξ
90	91	92	93	94	95	96	97	98	99	92	9ξ
80	81	82	83	84	85	86	87	88	89	82	8ξ
70	71	72	73	74	75	76	77	78	79	72	7ξ
60	61	62	63	64	65	66	67	68	69	62	6ξ
50	51	52	53	54	55	56	57	58	59	52	5ξ
40	41	42	43	44	45	46	47	48	49	42	4ξ
30	31	32	33	34	35	36	37	38	39	32	3ξ
20	21	22	23	24	25	26	27	28	29	22	2ξ
10	11	12	13	14	15	16	17	18	19	12	1ξ

3. From dozenal $2\xi 9$ ($D_{\text{bardozenal}}^J$, $J = \xi$) it is possible to find all its other D_{doz}^J and $D_{\text{bardozenal}}^J$ representations. The 1ξ results are given in Table 3 below.

4. In the polynomial form of any integer y (equation ξ), the leading (most significant) digit x_{t+1} cannot be zero: its sign is either positive ($x_{t+1} > 0$) or negative ($x_{t+1} < 0$), and its suffix, $t+1$, is either even or odd; call this its suffix parity. From the sign and suffix parity of the leading digit, and the sign of the base, it is possible to determine the sign of any non-zero integer expressed in D_B^J digits by consulting Table 4, which is true of all bases.

TABLE 3 : The 12 dozenal representations of one integer.

Base	J	Integer	J	Integer	J	Integer
+	2	229	7	1103	3	1103
+	2	219	6	1103	2	1103
+	9	1119	5	1103	1	1103
+	8	1103	4	1103	0	*
-	2	219	7	1103	3	1103
-	2	1119	6	1103	2	1103
-	9	1119	5	1103	1	1103
-	8	1103	4	1103	0	1103

* When $B > 0$ and $J = 0$, the recurring representation 12103 is unacceptable.

TABLE 4 : Leading digits and signs of numbers.

Positive base				Negative base			
Leading digit		sign	dozenal example	Leading digit		sign	bardozenal example
sign	suffix			sign	suffix		
+	even	+	(i)	+	even	+	(v)
+	odd	+	(ii)	+	odd	-	(vi)
-	even	-	(iii)	-	even	-	(vii)
-	odd	-	(iv)	-	odd	+	(viii)

Examples for Table 4

- (i) 207 ($4 \geq J \geq 2$) (v) 127 ($J = 1$)
(ii) 12 ($9 \geq J \geq 1$) (vi) 1195 ($2 \geq J \geq 9$)
(iii) 207 ($9 \geq J \geq 7$) (vii) 71354 ($J = 4$)
(iv) 12 ($2 \geq J \geq 2$) (viii) 1195 ($2 \geq J \geq 9$)

5. The extraction of the two square roots of bardozenal 15541 using bardozenal working in $D_{\text{bardozenal}}^2$ digits:

$$\begin{array}{r}
 1 \quad 3 \quad 5 \\
 1 \overline{) 15541} \\
 \underline{1} \\
 23 \\
 \underline{3} \\
 265 \\
 \underline{5} \\
 0
 \end{array}$$

$$\begin{array}{r}
 2 \quad 7 \\
 2 \overline{) 15541} \\
 \underline{2} \\
 1287 \\
 \underline{7} \\
 0
 \end{array}$$

The sign/suffix parity rules (Table 4) give $135 > 0$ and $27 < 0$, and then:

$$135 \times 135 = 15541 = 27 \times 27$$

and

$$135 + 27 = 0 = 27 + 135$$

6. When the base is odd, there is a unique least-modulus digit-family, for which $J = (|B| - 1)/2$. When the base is even, as for dozenals and bardozenals, there are two least-modulus digit-families, for which $J = |B|/2$ and $J = (|B|/2 - 1)$. Putting $|B|/2 = 10/2 = 6$, these digit-families have $J = 6$ and 5 (see Table 1). The dozenal analogue of Colson* decimal digits (D_{ten}^5) is D_{doz}^6 .

* * * *

Immediate extensions to the subject of this article could be: (1) addition and multiplication tables for dozenal and bardozenal digit-families; (2) graphs of division-algorithms; (3) digital sums of numbers represented by dozenal and bardozenal digit-families.

All juxtapositional dozenal digit-family representations of integers can be considerably extended to include, either as rational fractions, or as dozenal- or bardozenal-pointed fractionals, the elements of rational-number field Q , of which Z is a subset. Moreover, DAs can be used to obtain dozenal- and bardozenal-pointed fractionals and shew, for example, that:

$$\frac{25}{18} = \frac{187}{24} = 18'0825\bar{2}, \text{ in } D_{\text{bardozenal}}^6 \text{ digits.}$$

* * * *

*Colson News: Two-Way Numbers, 1984

Note also 'Two Way Arithmetic' J.Halcro Johnston (DSGB) and 'The Reverse Notation' by same author, Blackie, 1950

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Editor's note. Mathematicians categorize sets of numbers as follows:

- N the natural numbers for counting: 0, 1, 2, 3, ...
- Z the integers, which include negative whole numbers or 'bar' numbers:-2, -1, 0, 1, 2,.... and which contain set N as a subset (N consists of the positive integers).
- Q the rational numbers, of the form a/b , where a and b are integers. Q contains all fractions, ratios and integers.
- I irrational numbers, which exist, but cannot be written in rational form precisely. The square root of 2, and π , are examples of irrationals.
- R the real numbers, which include all the above sets.