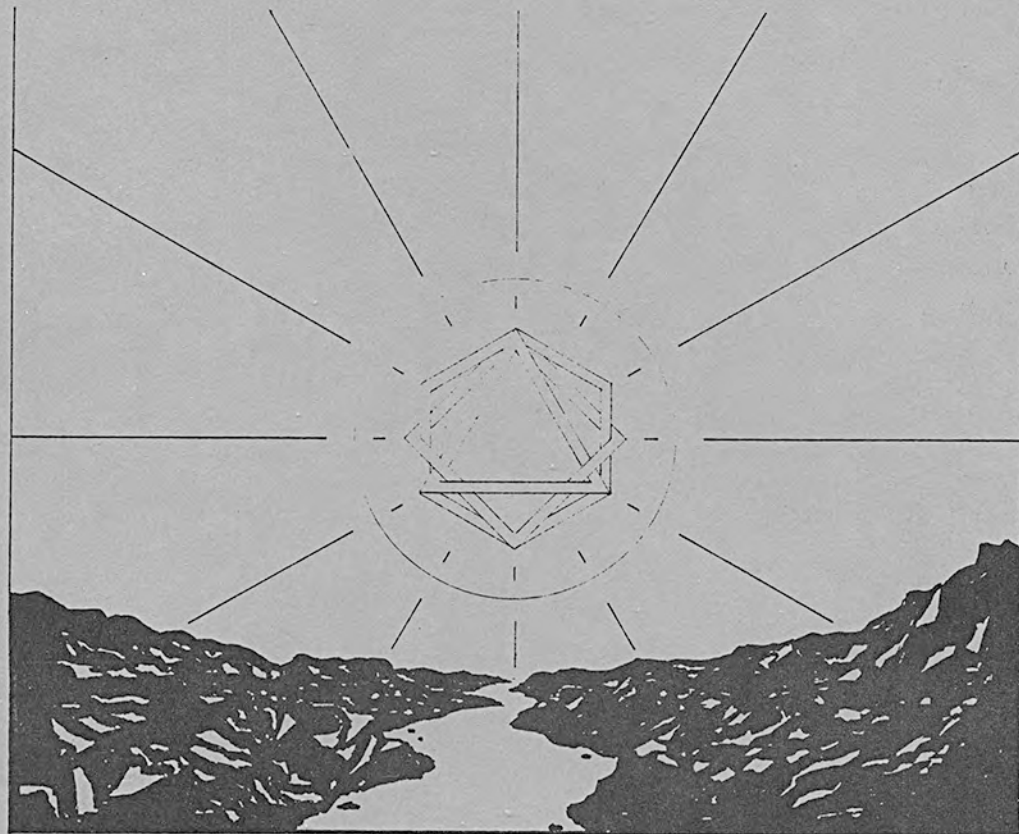


THE DOZENAL JOURNAL



NUMBER 8

SPRING

1197 / 1990

Published by:

The Dozenal Society of Great Britain
Denmead Hampshire England

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In association with the Dozenal Society of America
Long Island New York

ISSN 0260-4884

THE DOZENAL SOCIETY OF GREAT BRITAIN

This is a non-profit-making Society, with membership open to all, whose ultimate aim is the conversion of the base of numeration from the present decimal (ten-count) to a dozenal (twelve-count) radix.

It is also a prime concern of the Society that the mathematical, scientific and technical education of our children should be as broad as possible; should be freed from the political interference and censorship now prevalent in schools and public life; and should impart a sound understanding of number, with modes of arithmetic and measurement which are efficient and in harmony with the needs of mankind and the unalterable laws of the Universe.

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Articles, letters, cartoons, etc., are invited from all interested parties. The Editor reserves the right to modify contributions to suit available space, but will take care not to alter conclusions/opinions.

THE DOZENAL SOCIETY OF AMERICA

This has broadly the same objectives as above, and maintains a cordial relationship with the DSGB.

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Prof. Gene Zirkel
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EDITORIAL

THIS ISSUE IS EVEN MORE anti-metric than usual. No apology is made for this; but it may be observed that such sentiments are aroused as much by the insidious process of enforced metrication as by the second-rate qualities of the system itself.

Phased 'cut-off' dates arrive, like maggots emerging from an apple, to disfigure (in both senses of the word) our common measures and with them, step by step, our rational modes of thought. Implanted in the bodies commercial and administrative with little in the way of parliamentary debate, these metric maggots appear first as innocuous-seeming dual markings, such as "8oz (227g)". Who would quarrel with that?

We in Britain now know that it heralds a sequence which proceeds inexorably, at programmed intervals, along the lines of: 227g (8oz)...220g (7.8oz)...220g. At this stage, the politicians have done their job. The ounces have vanished, and the manufacturers can happily get on with shaving-down the grammes from the already strange number of 220 (Quick, now! What fraction is that of 1 kg?). Cans inspected in my larder carry weights of 340g, 410g, 450g, 300g, 185g and even 411g...

The decimalists' aim is the obliteration of rational measures for ever. Are Americans ready for the nibbling larvae? Will the United States resist infestation, or meekly submit to Sèvres?

* * *

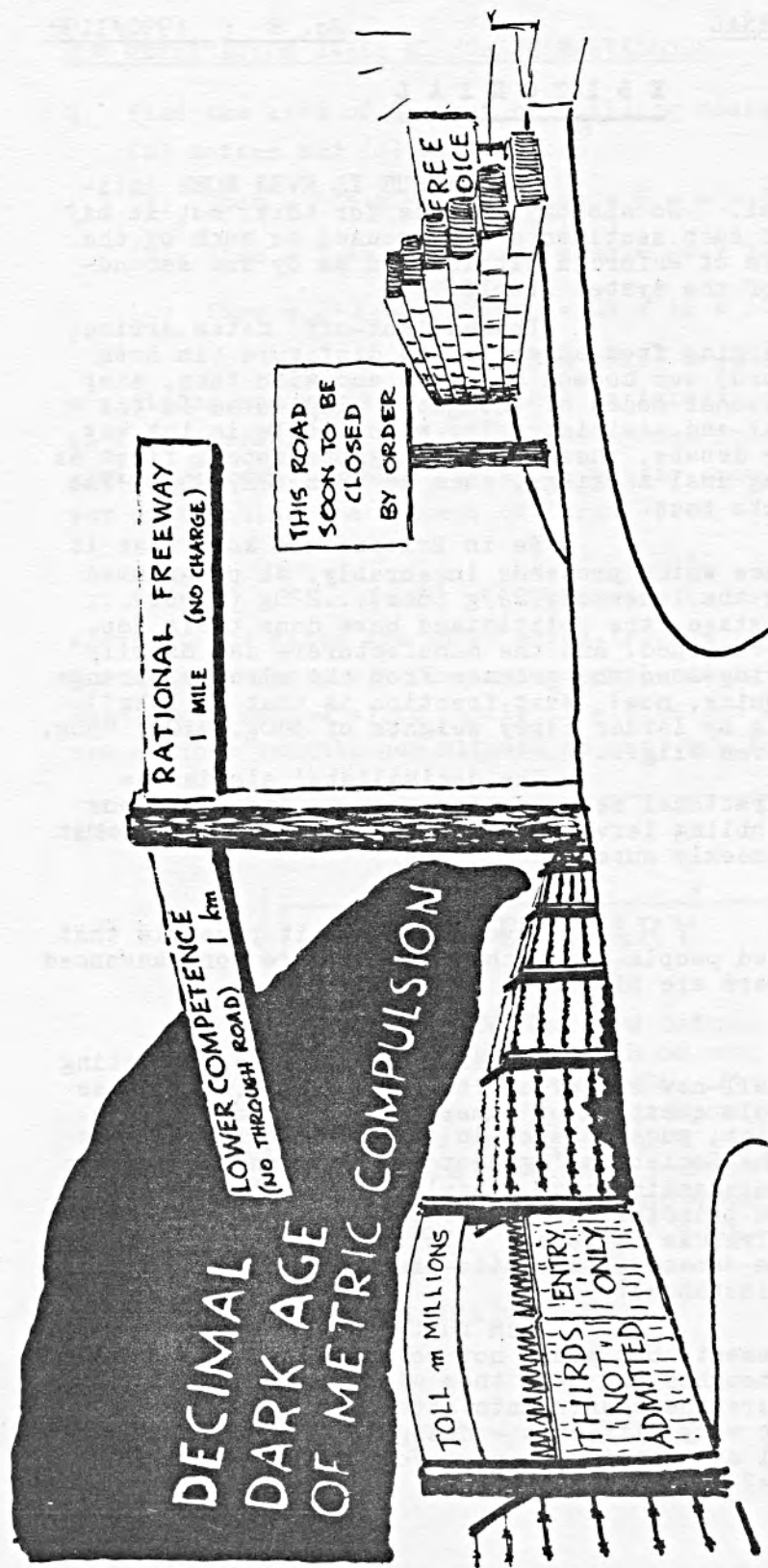
Psychology: is it possible that some simple-minded people think that metrics are more advanced because the numbers are bigger?

* * *

Australian dozenists, by opting for a virtually all-new set of digits (see later), may raise once more the whole question of numeration and nomenclature. It's a sticky point, much debated in the past. The present truce, whereby the Societies 'agree to differ' on these items (while fully understanding each other's language), preserves unity on the main principle of duodecimalism while we wait for consensus to evolve, as it were. Is it evolving? Perhaps we should re-open debate (I am quite prepared to do an article on 2 and 3, for instance)?

Such further debate might still not lead to agreement, but would not be wasted if it forced us to clarify our thoughts and back them with reasons. What do need exposition are the constraints attendant upon what is a fascinating - but very difficult - design exercise. This is something I shall attempt in the next Journal. Meanwhile, what do you think?

The Editor.



WHICH WAY NOW ?

The ROMANTIC

The English Magazine

WE HAVE BEEN taken to task over our note on THE ROMANTIC magazine in JOURNAL No. 7 for suggesting that their objection to metrication was no more than 'instinctive rejection of an alien imposition'. Miss P.M. Langridge writes: 'If the only issues were those of practical efficiencies then we should have no interest in the matter one way or another. It is because they are an attack on our historic culture that we are concerned about them; and I am quite certain that it is....for this reason....that they have been so ardently pursued.'

This contention is manifest in a companion publication, THE ENGLISH MAGAZINE*, which has a more analytical approach to the fallacies of present-day society. Its comments on these are quite refreshing, compared with the bland utterances from usual sources.

We do, however, recognize that the cultural aspects of weights and measures are of equal importance to the technical; the two are complementary. Traditional methods require to be forcibly removed before the less socially-acceptable forms can be made compulsory. Part of the process is to convince us that metric measures are 'more scientific' than those developed over the ages to be compatible with our physical needs and comprehension - which are dismissed as illogical and arcane. These certainly can be inconvenient for administration and planning** (as an admin. type remarked: 'I like metric; it makes things easy for me...')

Unfortunately, in the absence of knowledge as to why our vernacular measures are superior for general use to any committee-designed arrangement, many have to fall back on instinct for their objections. Part of our work is directed to counselling the public on the principles of calculation and measurement, so that opposition to needless and damaging change may have a sound foundation.

A.F.W

* £10-0-0 Subscription to Perfect Publications,
B.M. Perfect, London WC1N 3XX.

** Yet just see what an array of exact fractions can be found of a statute mile (and compare with kilometre) -Ed.

'Metricate in haste, repent at leisure'

IT is a pity that you included the metric system among the "good things" ascribable to the French Revolution; for the metric system is definitely not a good thing.

It was consciously designed to perpetrate the fundamental mischief of constraining measurement to arithmetic, instead of vice versa; resulting in an almost complete incompatibility with natural ratios and convenient, human-scale sizes. With its dismal, decimal rigidity it destroys practical and harmonious relationships, substituting needless four-figure numbers and clumsy approximations where single figures and simple fractions did previously.

As such, it is a culturally destructive weapon of great efficacy: it seems logical, to the

extent that it can seduce intellectuals, who can then be enlisted to assist in its imposition upon a bemused public. No wonder, then, that it was eagerly adopted, first by the Napoleonic hegemony which ruled Europe during the early part of the 19th century, and second by the Communists after the Russian Revolution.

It is indeed a measure of tyranny, as George Orwell noted; and the enforced displacement by it of Anglo-American rational measure, for wholly political reasons, will be a setback from which it may take centuries to recover. Metricate in haste, repent at leisure, indeed.

DONALD HAMMOND,
Secretary,
Dozenal Society,
Millside, Mill Road,
Denmead, Hampshire.

THE ABOVE LETTER was printed in THE SUNDAY TELEGRAPH of July 23 1989 (July 12 1199). Many letters of support (none against) were received from readers and we print selected quotations from some of them below.

"I feel very strongly that we are in danger of losing the very real advantages of all the special systems of measurement that have endured over hundreds of years to best suit the trades concerned.

Anyone with practical experience of these proven systems must do all in his power to retain them. The bureaucrats have no idea of the damage they are doing to man's accumulated learning by encouraging the teaching of the metric system as being the 'best' and 'only' system."

Jeremy Links
GOSPORT Hampshire.

"...I don't like the thought of going over to the metric system fully. I see now that they want to

take away the price in gallons from the garage fore-courts."

K. Davis
BRIXHAM Devon

"I am deeply concerned at the prospect of further metrication in 1992.....in spite of a government assurance that there would be no more compulsory metrication.

I see from your letter that you are Secretary to an organization called the Dozenal Society....I should welcome an opportunity to give it my support.

The arguments against metrication are legion and I am sure you know them all. One ray of light is the announcement by the Jockey Club that, unless ordered by the government to do so, they will not change to metric measures as this would make a nonsense of their records."

Major J.C. Ratcliffe
BRECON Powys.

"Further to your letter in the SUNDAY TELEGRAPH, could you please let me have details of the Dozenal Society?"

Richard M.J. Beere
BRISTOL

"I was surprised to see your letter. Over the last few weeks I have written similar letters to the press with no response. Indeed, since yours was the first I have seen with this point of view, I was beginning to think they were being suppressed!

I concur with everything you say but I feel that this system of measurement is just a symptom of an inhuman attitude to life which is positively evil. I suggest this because the attitudes of planners even in the fifties were showing the arrogance of the metric mind - we are infallible - this is how it is going to be - many years before the actual measurements were used. So.... we have seen buildings where.....people will not live and farm animals turned into cannibals because 'protein is just protein'....."

Keith R. Hodgson
CROSTON Lancashire.

"Although first mooted in 1790 and instituted by the Law of Germinal 18, Year III (April 7 1795), such was the reluctance of the French to use metric measures that further legislation was enacted in 1837 making them obligatory.

The kilo and the litre seem to have caught on fairly quickly, no doubt because of their proximity to established measures (two French pounds = 978 grammes; one French pint = 0.93 litre). But one has only to read nineteenth-century novelists to realize that leagues, fathoms, feet and inches persisted well into the latter part of the century, just as frequently in Zola (b.1840) as in Stendhal (b.1763), Balzac (b.1799) or Flaubert (b. 1821). Even when giving the depth of snow (La Bete

Humaine 1890) as 30 or 60 centimetres, Zola was plainly thinking in terms of one or two feet.

Given the slowness of the French to accept the benefits of their metric revolution, why should the rest of us be in any hurry?"

Ian Hamilton Walker
Edinburgh University.

"Does your Society support the Duodecimal System for counting, etc? I think it may have some merits..."

Dualta O'Gallacoir
Sandyford DUBLIN

"Not only do I dislike the way this country is always expected to do whatever other countries are doing, but... aspects of the metric system particularly rile me.

At school I was taught that one of its advantages was that you need never go above ten, as that brought another unit into play (even if they were much duller than our own units). Now we find hundreds and even thousands of those awful microbes called millimetres being freely used."

Hugh R. Fowler
LITTLEHAMPTON W.Sussex.

"...I have been pondering for some time the great practicality of the Imperial weights and measures systems and their appropriateness to the various scales of human activity, especially when coupled to the £sd money system. Compared to these the metric weights, measures and money seem to me to be abstract and inhuman in the extreme. I would be most grateful if you would send me some information about the Society."

R.S. Pitman
Barbican LONDON.

"May I please say.....how I agree with your views. After a dozen years of living in Switzerland I could not come to terms with this inhuman and meaningless system, and people there did still sell you a 'livre' of apples and spoke of 'sous' even though the actual values had been altered to fit the metric amounts. I don't know how it is in Hampshire, but here, although of course metric money has been thrust upon us, other measurements have for the most part quietly returned to the former ones.

My late father (Dr. J.C. Willis FRS, who was a botanist but mathematically inclined) always said that the metric system was a retrograde step, and that things should ideally be based on twelve.

I have no idea how it is explained to children that their ten cannot be divided by three, or what is the nature of a recurring decimal."

Anita Binnerts
RIPON North Yorkshire.

"Your letter struck an immediate chord. Over many years I have felt it to be perverse for the very reasons you give and many more could be added; but you have opened my eyes by giving the French metric system a political raison d'etre. Will you please send a little more detail of the DOZENAL SOCIETY?"

Russell Stokes
NOTTINGHAM.

"How can this government call itself 'Conservative' when it promotes the destruction of so major a part of our history and culture?"

Although I think we have been told that the mile and the pint are to be saved, how can we trust them an inch? I fear for our Rule of the Road as well... You have probably noticed how Radio 4's wildlife programmes have been rendered almost incomprehensible under a plethora of metres, centimetres and hectares."

Carol L.K. Gibbons
CROWBOROUGH Sussex.

"Congratters on your Telegraph letter.... I think that there ought to be a real anti-metric campaign. How much good it would do is open to doubt, but it should at least be there."

Priscilla Langridge
THE ROMANTIC Magazine.

"You take a line which interests me; my main concern is that of opposing the actions of bureaucrats in forcing us into a mould which we have not requested nor had any opportunity to express an opinion about...."

Graham Boatfield
IPSWICH Suffolk.

"...The last German Kaiser once stated: 'The success of the British is their system of measure.' Never a truer statement was made. In 1864 the metric system was rejected (for trade) by the British Parliament and from that date we could outstrip our Continental competitors in the manufacture of virtually anything.... ranging from darning-needles and safety-pins to railway locomotives and battleships..."

Angus MacDonald
LONDON.

"I have long been an opponent of metrication but have always felt I was a 'lone voice crying in the wilderness.' I was particularly pleased, therefore, to hear your views and that a Society opposed to this measure exists."

C.W.R. Cooke
AMERSHAM Buckinghamshire.

A forgiveably sarcastic note from a member rounds-off this section appropriately:

"I am able to buy Danish jam in pots clearly marked: '2 lb.(907gm). Seems nobody has yet informed them how difficult they find it."

Robin Hancock
FOWNHOPE Herefordshire.

STATUTE SERENDIPITY

A feature of traditional units that causes anguish to the metric brigade is how, so often, they happen to work together in a harmony which, according to metric thinking, should be impossible with such evolved sizes.

One of the best examples is that of vehicle braking distances. A vehicle is said to have '100% braking' if the braking force equals the weight of the vehicle.

So, taking the speed prior to braking as 30 mph (I am working in denary here), which is 44 ft/sec., we can use the formula:

$$v^2 = 2gs$$

where V is the speed, g is gravitational acceleration (brake force = weight, remember) and s = distance to stop.

To find s:

$$\begin{aligned} s &= \frac{v^2}{2g} \\ &= \frac{(44)^2 \text{ft}^2}{\text{sec}^2} \cdot \frac{\text{sec}^2}{2(32.2)\text{ft}} \\ &= \frac{1936}{64.4} \text{ft} = \underline{30.06 \text{ft}} \end{aligned}$$

So, to a very high degree of accuracy, "perfect" brakes will stop your vehicle from 30 mph in 30 feet. Rather nice, that.

* * *

ALL THE THREES

Still staying with ordinary arithmetic and everyday measuring, here are on-the-spot quick conversions for general estimation or DIY.

Rough approximations can be obtained as follows.

Feet to metres: MPLY by 3 and DVD by ten, e.g.

9ft gives $9 \times 3 = 27$, or 2.7m.

Metres to feet: MPLY by ten and DVD by 3, e.g.

1.7m gives $17 \div 3 = 5\frac{2}{3}$ ft, or 5ft 8in.

As we said, rough - but sometimes useful, remembering that dress and furnishing fabrics must by law be supplied to the decimetre. Handy for timber estimates.

An accurate conversion from metres to feet and inches can be done mentally for whole metres or half-metres. Multiply by 3 three times: once for feet, once for inches and once for eighths, as below.

metres		feet	inches	eighths	
1	x3 =	3	3	3	= 3ft 3 $\frac{3}{8}$ in
2	x3 =	6	6	6	= 6ft 6 $\frac{3}{4}$ in
1.5	x3 =	4 $\frac{1}{2}$	4 $\frac{1}{2}$	4 $\frac{1}{2}$	= 4ft 11 $\frac{1}{16}$ in

This is accurate to within 0.005 in. per yard (99.98% accurate).

Metric Muddle

Beryl Wilkins

School rushed feet first into teaching metric
but the outside world is still yards behind..

EIGHTEEN YEARS have passed since teachers were advised the UK was going metric, but confusion over what to teach continues.

In the teacher's manual of a text book intended for first-year juniors, published in 1971, it says "A recommendation has been made to adopt the metric system However, it is believed some time may elapse before it is fully operative."

Today we still teach a hotch-potch. The notes went on to advise "It is likely that children will continue to meet in their everyday lives certain of the Imperial units....to avoid confusion the two systems should not be taught side by side."

This dilemma is difficult to avoid. Text books, workbooks and maths. equipment are based on the metric system while the children's experience out of school is in Imperial units.

The current policy, throughout the age range, of primary schools known to me is to teach only the metric system but to refer to the Imperial where relevant. This seems to work and the duality is accepted by children (they've known nothing else), but there are worrying factors.

The most common difficulty seems to be where the child tries to compare something s/he has come across outside school (described in Imperial units) with something s/he has discovered in the classroom (using the metric system). For instance, my class of seven- and eight-year-olds measured one another and made a chart. Andrew, height one metre 30 centimetres, said next day "My Dad says I shall grow as tall as him."

"How tall is your Dad?"

"Six feet, one inch."

We did a calculation to find out how many centimetres he needed to grow. But until this is done - or a conversion table is used - there is no way the child can visualize an immediate comparison. A similar thing happened in a top infants' class. The pupil was reading to her teacher "My baby weighs four kilos; he is very light." The little girl stopped reading and said "Our baby was eight pounds and four ounces. Is that heavy?"

Where parallel systems are used they are almost certain to get no more than a vague idea of the one which is least familiar to them.* Do we, if we come across the well-known lines: "Full fathom five thy father lies ..., visualize anything more than "pretty deep"?

Perhaps for some children, there

*Or is more awkward to use? -Ed.

is no cause for anxiety, but for others the problem is already too much for them to cope with. A colleague who teaches a class of first-year juniors with special needs (not necessarily low IQs) has, after much experience and thought, abandoned the metric system altogether when teaching weight. These children already experience great difficulty with maths, language and the only everyday experience they get involving weight is when they go down to the corner shop for three pounds of potatoes, or big sister loses half a stone when she goes on a diet.

Imperial measurements seem to be the natural expression when it comes to creative writing. A seven-year-old describes in his Underground Adventure "I came to a molehill as big as a pond. It was 17 feet wide and 17 feet long....it must have been about a hundred and eleven feet deep."

And that from a child who had recently completed a page in a workbook which required him to measure the height of the door, length of the classroom, width of his table and so on, in metres.

Conversations with local tradespeople suggest that not only is the process towards complete metrication at a standstill but, in some cases, there is total reversion to Imperial units. Is there really any justification for teaching in metric when signposts indicate in miles, speed is registered at miles per hour, petrol consumption is calculated at miles per gallon and milk is delivered in pint bottles?

The compulsory change to decimal currency was swift and complete. Isn't it time a short, sharp shock was applied on behalf of total metrication? Or should the UK scrap the idea altogether?

* * *

The above article first appeared in TEACHER magazine in 1989 and is reprinted here by permission.

* * *

Imperial measures

Timber merchants James Latham are returning to feet and inches after nearly twelve years. The timber trade was one of the first British industries to use metric measurements in the early 70s. To make the changeover easier,

the Burnikell rule was devised: it was 39.4 ins (one metre) long, and was split into 33 divisions, giving the illusion of a yard. This was found to be more than a little confusing by customers. New technology has now provided a method by which a tally can be provided in the customer's choice of measurements.

WOODWORKER MAY 1990

GOOD NEWS! But thirty-three divisions?

Some mistake, surely....?

PENTAGONAL PROTRACTION



Troy

SOMETIMES, when advocating the use of the dozenal base by pointing-out that it will accept three as a factor and can therefore cope properly with thirds, I have evoked the retort: '...but then you can't get fifths exactly!' from people who are not only perturbed by the loss of five as an 'easy' factor (in decimal count), but also prone to think that one prime factor is as good as another; that to gain three and lose five is merely a matter of taste and so certainly not worth the disruption involved in changing the base of arithmetic.

Now, this is a serious impediment to our case. Of those who have even heard of us many, particularly if they have not taken the trouble to investigate the workings of arithmetic and geometry, have a mental image of dozenists as either madmen (for wanting to change the numbering system) or reactionaries (for not wanting to lose the tradition of rational measurement). Those of us lucky enough to have received our basic education in more enlightened times, when multibase arithmetic was the norm and counting-up in, say, sixpences was something everyone could do, sometimes tend to forget that there is now a whole generation of people to whom the half-base count of five, ten, fifteen, twenty,..., is the only variant known; to whom the loss of five as an auxiliary base would seem incomprehensible and wanton. These, in their impressionable years, have been and are being moulded by the State - via its willing servants in education - into thinking 'five-and-ten' exclusively, when counting or measuring. A youngster of today will quite confidently render twice £3.50 as £7.00, but if asked for twice 3ft 6in is liable to answer '6ft 12in' or even '7ft 2in'.

The statutory decimalization of thought is driven by politics and fashion: there is no way by which the contemptuous rejection of three as a principal prime factor can be justified otherwise*. We dozenists must not

*See DOZENAL JOURNAL No. 6 for a dissertation on three.

fall into the same trap: our resistance to five as companion prime to two must be firmly based on merit, not taste; and to this end we should examine the properties of five carefully, so to determine just how it signifies in the hierarchy of numbers.

* * *

If we amuse ourselves with the ancient exercise of arranging numbers as points on surfaces or in space, we find that one is dimensionless, two is a LINE-number (1 dimension), three is a TRIANGLE-number (2 dimensions) and four is either a SQUARE number (2 dimensions) or a TETRAHEDRON number (3 dimensions). Thus, the first four numbers alone are sufficient to take us to the maximum spatial dimensions we can realize in practice; so illustrating the fundamental importance of these small numbers and why dozenists are insistent upon a radix which accepts them properly.



TETRAHEDRON:

Any two define a line;
Any three define a plane;
All four define a space.

The tetrahedron, like the triangle, is inherently rigid.

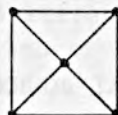
The disposition of points in space as vertices of solids is something perhaps better explored in a separate article; here, however, we may remark that if we are seeking shapes with order and symmetry, all prime numbers - except two and three - will define PYRAMIDS, but little else. Such pyramids in plan yield centre-pointed polygons.

7



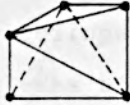
The tetrahedron (above) is, of course, a pyramid; so four is the first pyramid-number. Four is not a prime number. However, if we use four points in a plane to form a square, then a fifth point outside the plane can give the square pyramid of the Egyptians. The plan view

5



shows the five points in their traditional representation as a centre-pointed square, found on dice and dominoes. Five is thus the first prime number to form a pyramid: it is the first 'pyramid-prime'.

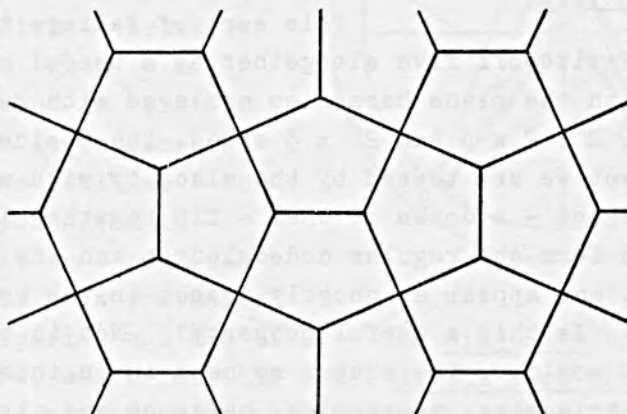
Five points may also be disposed as vertices of a triangular dipyrmaid (a paper version is easy to make) which has six faces and nine edges.



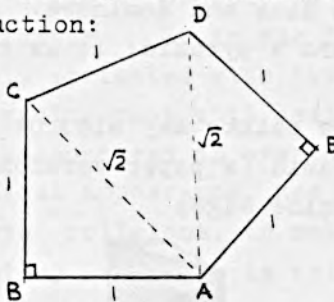
This interesting shape, being, as it were, two tetrahedra joined at one face is not a regular polyhedron; it is, though, an inherently rigid frame. Not of much use in buildings, perhaps, it might commend itself to the constructors of space-stations: such a frame, built from nine straight beams, would need no internal bracing. There seem to be just three rigid frames: triangle, tetrahedron and this hexahedron, with numbers of vertices 3, 4 and 5; but I make no further comment on that at present....

PENTAGONS

Five points on a plane surface will mark-out an infinity of pentagons, even if we specify all sides to be equal: the pentagon is not inherently rigid and can be distorted freely while remaining equilateral. This property was exploited by P.A. MacMahon (New Mathematical Pastimes, CUP) to design the exceptionally elegant tessellation of equilateral (not regular) pentagons given below.



It will be noticed that MacMahon's pentagonal element has two right-angles. Here is its construction:



Construct $\triangle ABC$: $AB = BC$;
 $\angle ABC$ is a rt. angle;

Construct $\triangle ACD$: $AD = AC$;
 $CD = AB$;

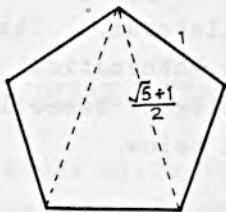
Construct $\triangle ADE$: $AE = DE = AB$.

ABCDE is required pentagon.

Concrete units cast in such a mould should make an impressive garden pavement, perhaps.

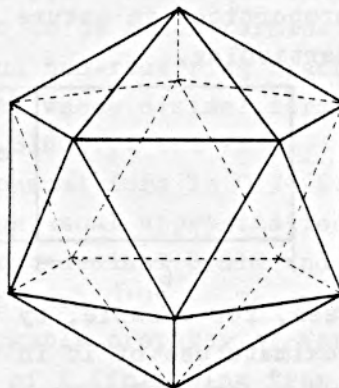
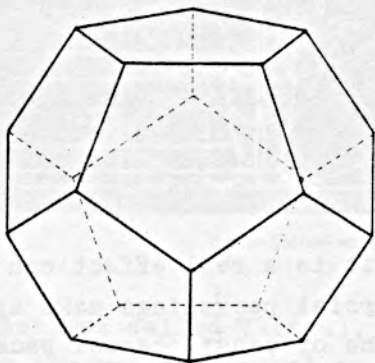
REGULAR PENTAGON

Just as the equilateral triangle expresses 'three' and the square characterizes 'four', so it is the regular pentagon which properly depicts 'five'. This figure will not form plane tessellations with itself or with other regular figures. In order to generate the tessellable pentagon above it is necessary to adopt a side : diagonal ratio of $1 : \sqrt{2}$ (stolen from the square), whereas the side of the true pentagon bears the more complex ratio $1 : \frac{\sqrt{5} + 1}{2}$ to its diagonal. The interior angles resulting from this ratio prevent plane tessellation.



This sort of failure tempts us dozenists to write-off five altogether as a useful prime: by comparison with the plane harmonies achieved with regular figures of 3, 2^2 , 2×3 and $2^2 \times 3$ sides, the 5-side performs lamentably; yet we are teased by the alacrity with which regular pentagons - a dozen of them - fit together in three dimensions to form the regular dodecahedron and the great dodecahedron, and appear as ghostly frames in the regular icosahedron. Is this a useful property? Not in the everyday practical world. The shapes we need in engineering and building are triangles, rectangles, hexagons and circles for

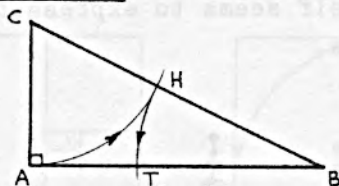
the most part; one sees few dodecahedra about the streets. Yet these figures exist; they are beautiful expressions of spatial truths; they make us think. The regular icosahedron, in the elevation given, displays the profile of a regular hexagon, faces which are regular trigons and edges by which twelve intersecting regular pentagons may be discerned (and which are made explicit in the great dodecahedron).



The significance of five here is that the regular pentagon is a limit: the maximum number of sides possible on each face of a regular polyhedron is five.

* * *

FIVE & THE GOLDEN SECTION

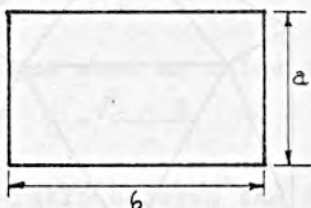


If a right-angled triangle ABC is drawn so that the two sides containing the right-angle are in ratio 1 : 2 ($AB = 2AC$), an arc with radius equal to the shorter side AC and centred at C cuts the hypotenuse at H. If a second arc is now drawn with radius BH, centred at B, to cut the base-line AB at T, then AB is divided at T in Mean and Extreme ratio: the ratio of the smaller part to the larger equals the ratio of the larger part to the whole line.

$$\frac{AT}{TB} = \frac{TB}{AB}$$

This ratio is the Golden Section.

A rectangle whose adjacent sides are in this relationship has, since its discovery by the Greeks, been held to embody the most pleasing and harmonious proportions possible. Other rectangles seem to be too fat or too thin for a satisfactory appearance; the Golden rectangle is just right. There are deep psychological reasons for this, which are not fully understood but which may have some connection with certain perceived proportions in nature generally and in the human body in particular.



GOLDEN RECTANGLE

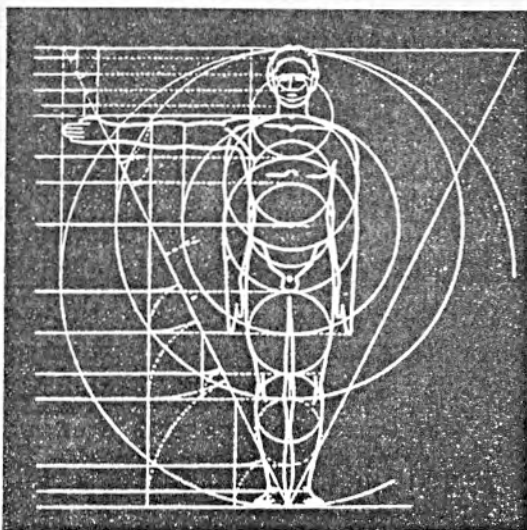
$$\frac{a}{b} = \frac{b}{a + b}$$

That it is a real effect can be seen, for example, by the way commercial packagings make approximate use of it in the proportions of cans*, cereal packets and so on. The humble postcard is this shape, as are many national and state flags (the British, however, favour a 1 : 2 ratio with flags). Architecture, from classical Greek temples (but not the Parthenon) through the Renaissance to modern times, uses the Golden ratio in plans, in elevations, in features such as windows and building modules; many great paintings show it subtly in their compositional distributions and the human body itself seems to express the proportion in its relative dimensions.

Golden Section in the Tempietto of Donato d'Angelo Bramante (1502). The rectangles over the drawing of the round temple are 'golden'. The Tempietto, a pure central building, stands in the monastery courtyard of S. Pietro in Montorio in Rome



*See DOZENAL JOURNAL No. 3, p. 2.



Golden Section and the human figure

Numerically, the Golden Section can be found only approximately: like the diagonal of a square, it is one of those ratios which forced the Greeks to accept the existence of irrational numbers. A simple calculation shows it to be a non-terminating and non-recurring fractional whose decimal form is 0.61803... and whose dozenal form is 0'74ĒĒ... The usual approximations are: decimal 0.618 and dozenal 0'75. The value,

sometimes called τ (tau), has the remarkable property of forming its own reciprocal by the addition of 1 (following from the definition): $\tau + 1 = \frac{1}{\tau}$ and this equation can also be used to calculate the value. Furthermore, the Golden rectangle divides into a square and another (smaller) Golden rectangle; a repeatable process which can lead to approximation of a logarithmic growth curve of the type found in snail shells and flower seed arrangements.



Brief mention must be made of the Fibonacci Series, discovered by Leonardo of Pisa (in connection with the breeding of rabbits) in the thirteenth century, where each successive term is the sum of the preceding two terms: decimally 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... dozenally 0, 1, 1, 2, 3, 5, 8, 11, 19, 2Ē, 47, 75, 100, ...

This series is found to control growth patterns in plant leaves, seed heads, fir cones, etc.

The reason for its inclusion here is that successive pairs of terms in the Fibonacci series approximate more and more closely to the Golden Section; and in practice the ratio 5 : 8, or the fraction $\frac{5}{8}$, is taken often as a reasonably near value for it. British 'foolscap' writing-paper measures eight inches by thirteen, which is even closer. (Readers wanting further detail in these matters might consult 'Practical Applications of Dynamic Symmetry' by Jay Hambridge, Yale Press 1932, or numerous other similar works; and for school exercises, 'Mathematics through Experience' by Holt & Marjoram, Blond Educational, 1968.)

The point which concerns us here emerges when we calculate the value of τ in surd form:

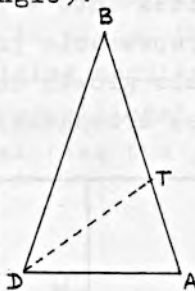
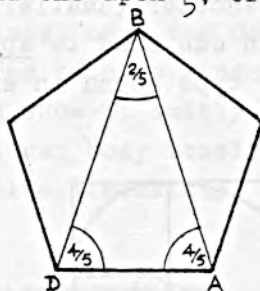
$$\tau = \frac{\sqrt{5} - 1}{2}$$

and

$$\frac{1}{\tau} = \frac{\sqrt{5} + 1}{2}$$

—and recall

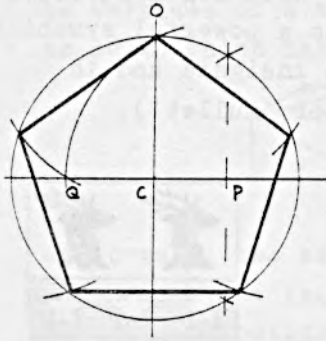
the ratio between the side of a regular pentagon and its diagonal given earlier. Euclid discovered the construction of the regular pentagon (Book IV, Prop. 11, et seq.), having first deduced that the isosceles triangle formed by a pair of diagonals and the included side has the property that each base angle equals twice the apex angle (i.e. each base angle is $\frac{4}{5}$, and the apex $\frac{2}{5}$, of a right-angle).



ABD is the isosceles triangle in question, and it follows that if such a triangle can be constructed, then so also can the regular pentagon. It is not difficult to show that, if DT is drawn equal to DA, the three lines BT, DT and DA are equal; and that T divides AB in the Golden ratio. Hence, this special triangle can be constructed by first drawing the side AB, dividing it in mean and extreme ratio at T by the method shown earlier, then completing the triangle by locating D (TD = AD = BT).

This was the method used by

Euclid to construct the regular pentagon; it shows clearly the intrinsic relationship with the Golden Section. An elegant method, derived from Euclid's, allows swift construction in a given circle. This appeared in DOZENAL REVIEW No.30, and is reproduced here.



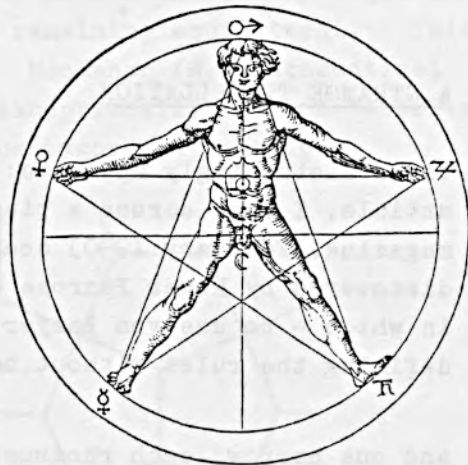
CP is half the radius of the given circle. If, for the sake of simplicity, we call the radius 2 units, then $CP = 1$ and the second radius $PO = \sqrt{5}$. This makes CQ equal to $\sqrt{5} - 1$ and in Golden ratio to CO , the original radius. OQ is then the length of side of the pentagon. (CQ and CO are two sides of a Golden rectangle and so

this is another method of drawing that shape, too.)

A CHANGE OF PLANE?

Five's a number we have seen
In Magic signs at Hallowe'en...

The re-entrant regular pentagon, or 5-pointed star, is an attractive figure which has been imbued with magical properties for many a century. With a vertex at the top, it is 'good' - a life symbol - but is 'bad' if inverted, when it represents the face of the Evil One. The pentacle, or pentagram is - if I may put it so - regarded in some circles as a protection against evil spirits and the like. The diagonals of the convex pentagon, which form the star, divide each other in the Golden ratio as well as bearing that ratio to the sides of the



pentagon; and it may be that this psychologically attractive and 'organic' proportion accounts in part for the figure's popularity. It is the first and simplest of the star shapes, easily scribbled with five straight lines, and it is not surprising that this device - of little practical use and so not encountered in everyday structures - with its decorative, mystical appearance, has been employed as a powerful symbol in 'Earth' religions, in national flags and insignia and in heraldry (where it is called a 'molet', or 'mullet').



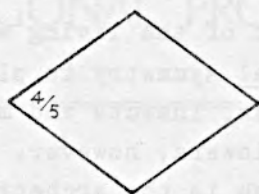
On the left is a witchcraft symbol; on the right, a school badge, part of which is described as '...a fesse vert with three molets of the field..'

* * *

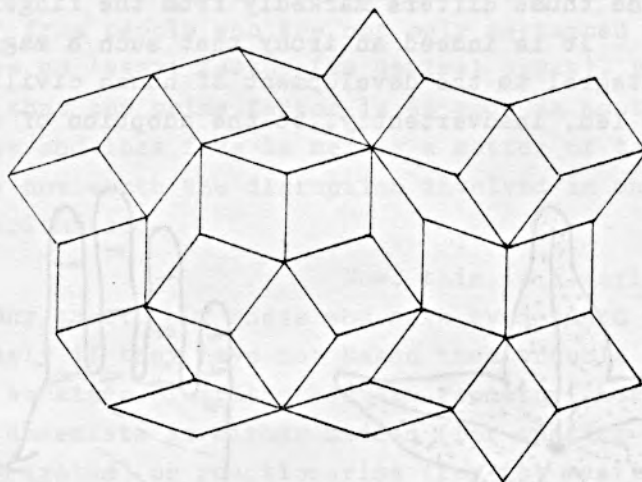
A STRANGE TESSELLATION

The workings of chance are sometimes disturbingly apposite: having got thus far with this article, I came across a piece by Hans von Baeyer (DISCOVER magazine, February 1990) concerning a decidedly odd tiling discovered by Roger Penrose (a British mathematical physicist) in which - to use von Baeyer's words - '...pentagons were defining the rules without being actually present.'

Two rhombi are used, one broad and one narrow: each rhombus features angles found in the regular pentagon and its diagonals (given here as fractions of a right-angle). These may be fitted together to cover a plane in a quite remarkable manner; one which, to my mind, shows to perfection the character of the number five.



Here is a small area of the resultant tiling.



Now, this tessellation is neither regular nor semi-regular. It is not even periodical: the patterns recur, but not uniformly. The description used is 'quasi-periodical', which seems about right for something undeniably perched on the borderline between order and chaos; patterned, yet no more than a step from randomness. There is a hidden order to it, which can be revealed by superimposing a five-directional grid of lines spaced not equally but at two distances according to.....The Golden Section.

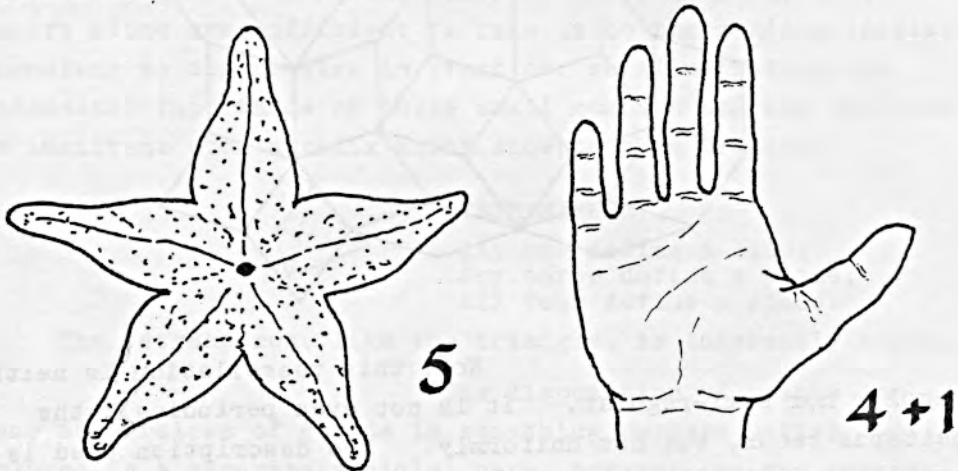
The principle can be extended to three dimensions*. It seems, then, that to tessellate 'five-wise' we must either (a) make each tessera five-sided, but take angles from the square; or (b) use tesserae which have pentagon angles, but only four sides.

*It is possible in theory to fill a space with three-dimensional rhombohedral elements to give a lattice of 'quasi-crystals' which could display fivefold symmetry - impossible with real crystals - when subjected to X-ray analysis. Some metallic alloys have been found which do just this.

LIFE

What of the living world?

Nature limits the use of rotational symmetry to plants and marine animals: land mammals, birds, insects and molluscs are bilaterally symmetrical. Many flowers, however, display pentagonal arrays; and the starfish is the archetypal five-animal. The human hand - responsible for the denary counting system - is not, strictly speaking, fivefold: in its structure the thumb differs markedly from the fingers which it opposes. It is indeed an irony that such a magnificent feature, integral to the development of human civilization, should have led, inadvertently, to the adoption of an impeded arithmetic.

SO, WHERE ARE WE?

This has been an exercise in being fair to five, in the hope of establishing its place in the hierarchy of prime numbers. Dozenists have always averred that priority should be given to three as a companion prime to two; a view which I strongly support as being an imperative condition for arithmetic and measurement. In previous articles (JOURNALS 4 and 6) I positioned five at the surface, as it were, between clarity and obscurity; and I see no reason to alter that description.

As I believe J. Halcro Johnston said: 'The decimal system goes in twos and fives; everything else goes in twos and threes.' Adoption of the dozenal base

rejects five as a mainstream factor, twelve being prime to five, which is as it should be. We cannot incorporate three prime factors in an everyday, practical number-base if it is to be easily manageable.

Among prime numbers, two and three are the natural workhorses of practical arithmetic and mathematics whereas five - to continue the metaphor - is a sort of Unicorn: elusive, difficult to handle, yet exerting a discernible influence in the more recondite regions of two-dimensional and particularly three-dimensional perceptions, where it seems to signify a limit of some kind. It is the last prime for which we have a rigorous plane division of the circle. It is more magical than mundane, more subtle than serviceable; yet we must recognize its presence appropriately if we are to have a properly flexible system, capable of expressing mathematical facts as readily and simply as can be.

The main point is that five becomes significant as a factor only when we start measuring angles. The dozenal base has prime factors two and three, which are the essentials: calculation and linear (hence also square and cubic) measure need a base which contains both; without them both, unnecessary clumsiness and incompatibility obscure what should be clear natural ratios and numerical patterns. With angular measure, 2 and 3 are still paramount; but I think we can see that in this field five ought to be accommodated if possible.

Fortunately, it is possible.

The Sumerians had a similar problem (though the other way round): their basic count was denary, but they came to see the need to have three as a factor; so chose sixty as a secondary base. The natural angle obtained when a circle is stepped-off by its own radius (sextant) was divided into sixty degrees, which led to ninety degrees for the quadrant.

We can use a 'Sumerian' solution with angular measure to accommodate five by dividing our right angle into five dozen parts (I call them 'rates'). This gives a 'round' figure for the quadrant in both dozenal and denary scales. It has the additional merit of being constructible in the plane (ninety is not) and gives exact results when divided by 2, 3 or 5. This 'rational' protractor is equally

factorable overall with the present one, despite being easier to read (1 rate = $1\frac{1}{2}$ degrees, or $\frac{1}{4}$ minute on the clock-dial).

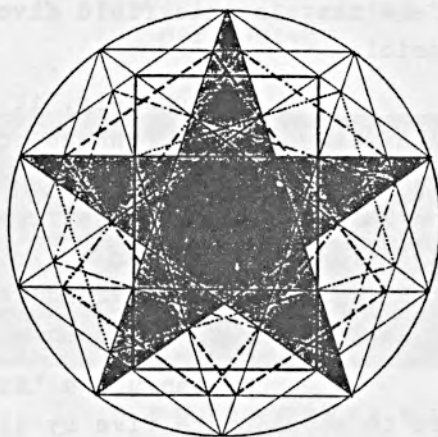
We must avoid, as I think the article has shewn, the exclusion of five as a factor of the angular scale (it is very tempting to design a scale of six dozen to the quadrant) since that would lose us both competence and credibility. Note, for comparison, that the decimalists intend to drop three as a factor in their Grade device, which is the foundation of the metric system.

We are much better than are the decimalists at arithmetic and measurement, having incorporated the sturdy factor three as a fundamental prime. In so doing, we have freed the unicorn-like five from practical work for which it was never suited. We should not lose the services of this capricious number, however, and its retention in a secondary, angular base will ensure its assistance when and where it is needed.

* * *

Sources not mentioned in the text:

Styles of Architecture: Herbert Pothorn, Batsford, 1971.
 Mathematical Models: Cundy & Rollett, OUP, 1961.
 The Witches' Way: Janet & Stewart Farrar, Robt. Hale, 1984.
 Encyclopedia of the Occult: Fred Gettings, Rider. 1986.
 Riddles in Mathematics: E.P. Northrop, EUP, 1958.



Five golden rectangles produced by the intersection of two superimposed pentagrams

E D U C A T I O N A L C O M M E N T S

THOUSANDS OF POUNDS are being spent in state secondary schools on computers. They seem to be everywhere, with whole classes engaged in staring at VDUs and the pocka-pocka-pocka of keyboards filling the air. What are they really learning? Since the advent of these devices in schools I have not become aware of any new flowering in mathematics (or any other school subject).

What I do find is Cambridge University's Engineering Faculty announcing that it needs an extra undergraduate year to make up the mathematics that was previously taught at school.

* * *

Robin Hancock reports that the question: "How much is a gross?" failed to elicit any answers from children in a TV quiz show... Which seems all-of-a-piece with the infants' counting lesson on schools TV which suggested that 11, 12, 13, etc. should be called: "Onety-one, onety-two, onety-three..."

* * *

THE NATIONAL CURRICULUM in mathematics is about to be implemented in schools, for all pupils. It embodies specific Attainment Targets with, needless to say, ten levels of attainment for each A.T. It must be said that the standard aimed for at the top level is very good: well up to GCE 'O' Level generally. There are some criticisms, however.

Number-Bases, as a topic, is out. Just as with GCSE (out not GCE), this study has vanished completely. Not even 'binary' remains. This pusillanimous omission comes as no surprise, now that even mathematics is being politicized...

Tables, as we used to call them (now known as 'multiplication facts') are to proceed only as far as ten-times-ten. Same comment as above.

Conversions from fractions to decimals and percentages will be taught, but not vice versa.

Geometrical distances and angles will be treated by wholly trigonometrical methods (no surd ratios).

Imperial Units in daily use may be taught only as whole units: no subdivisions to be used (e.g. miles will not be translated into yards or feet). Metric equivalents must be learned.

* * *

Although internationally accepted as a sexagesimal 'point', the colon (:) is almost never used by teachers when they write clock times. They instead use decimal points, so that 10:25 is given as 10.25, and complain when children get wrong answers in time calculations.

* * * * * * * *

THE 10,000-YEAR-OLD INCH

Nigel Corrigan

ALTHOUGH THE METRIC SYSTEM was devised to simplify calculations it may well represent a major blunder by the Frenchmen responsible. It seems that a brilliantly simple system existed well over 10,000 years ago; one which related not only to the size of the Earth but also to mathematical constants such as Pi.

In comparison to the arbitrary metre, ancient measures may have been perfectly 'natural', embodying the two most important aspects of any logical system of measurement: 'natural' time and distance. Within this system the fundamental sub-unit, and the one which linked mathematical constants with natural aspects appears to have been the Inch, now relegated to 2.54 centimetres.

The Inch: a Natural Unit of Measurement

Metrickation of measurement was initially proposed as a means of simplifying calculations. The basic unit, the metre, at first sight seems to have many advantages. One metre is 100cm. or 1000mm. Very simple calculations of addition or subtraction are indeed made easier but the metre has a major weakness: it is an arbitrary length which doesn't relate to any natural measurement.

As a consequence there can be no logical relationship between two or more 'natural' functions, such as distance and time, and any relationship to mathematical constants could not possibly exist.

The French, who originated the metre in 1792, decided that any basic unit of measure should, 'naturally', be based on the size of the Earth. Unfortunately the surveyors and mathematicians, rather than using the Equatorial circumference, chose the distance from the Equator to the North Pole.

If the Earth were a perfect sphere then the metre would be a natural unit of measure. The Earth is not a sphere: the Equatorial circumference is some forty nautical miles greater

than the Polar. The original metre was defined as one ten-millionth of the distance from Equator to Pole but, as no allowance was made for the flattening at the Poles, this was useless for navigation. As recently as 1960 the metre was re-defined (for the third time), and became 1,650,763.73 wavelengths of the reddish-orange light emitted by the isotope krypton-86 in vacuo. *

In contrast to this arbitrary measure, the Inch has a natural harmony not only with time and the size of the Earth but also with geometrical constants such as Pi.

The basis of this proposition is another unit of measurement, supposedly taken from the length of an average man's forearm, the Cubit. It is this unit which is accepted as having been used in the construction of the Pyramids.

Reference books on the subject agree on one point: the Cubit was 'about' 18.25 inches. It is now suggested that the Cubit was 18.24 inches, precisely.

Evidence to support this is on view in Europe, ironically in Paris. The Obelisk of Luxor, now standing in the Place de Concorde, was presented to the French by the Viceroy of Egypt in 1831. Weighing over two hundred tons, the monument was carved from a single piece of red granite at about the time the Pyramids were constructed. It is reasonable to assume that the stonemasons involved would have used the standard measuring-unit of the time.

Today, we see this obelisk as being 76 feet tall, but how did the stonemasons see it? If the Cubit is indeed 18.24 inches, then the monument was designed to be $76 \times 12/18.24$, or 50 Cubits precisely. A reasonable figure to select, one might suppose, when the structure was still 'on the drawing-board'. Pyramids were also designed using the standard Cubit. Experts are undecided on the original dimensions of the Great Pyramid of Cheops. We know what they are today, after thousands of years of erosion, but what did the original plans show?

The base of the Pyramid measures 756 feet, or 9,072 inches or 497.3 Cubits. It would be reasonable to consider that the

* How anyone can talk seriously in this context about 0.73 of a wavelength of light leaves me baffled. -Ed.

original base-length was 500 Cubits, and that erosion has removed only 2.7 Cubits (less than 17 inches per side). This supports the proposition that the dimensions of the Pyramid are accurately related to the size of the Earth: addition of base dimensions gives 2,000 Cubits or 36,480 inches, which is precisely one-half nautical mile, or 30 seconds of arc at sea level.

Consider the Cubit in relation to the size of the Earth: the (equatorial) circumference is 21,600 nautical miles, or 1,575,936,000 inches (an impractical figure); but what if the unit of measurement was the 18.24-inch Cubit?

Divide 1,575,936,000 inches by 18.24 and we have 86,400,000 Cubits, a number which will immediately be recognized by professional navigators as the number of thousandths of seconds in a 24-hour day. As the Cubit appears to have been defined by the predecessors of today's navigators, this should come as no surprise!

Fortunately for modern-day navigators the sexagesimal, or base-sixty system has survived within their field. 360 degrees, or 21,600 minutes, or 1,296,000 seconds still make up a full circle, as do 60 minutes an hour and 60 seconds a minute. As navigation is simply the relationship of distance, direction and time, this arrangement is perfectly logical; but how can the Cubit relate to both time and distance in easily-managed numbers?

If we accept that the circumference of the Earth is 86,400,000 Cubits, then:

1 degree of arc = 240,000 Cubits and 1 minute = 4,000 Cubits - which is of course the one nautical mile used by navigators today; but as a sub-unit, 4,000 Cubits perhaps makes more sense than 6,080 feet* or decimal fractions of the same**.

In Solar time, 24 hours results in the passage of 86.4 million Cubits on the surface of the Earth below a vertically overhead sun. One hour would see the passage of 3,600,000 Cubits, one minute 60,000 Cubits - and one second, 1,000 Cubits.

Thus, the apparent passage of the sun over the Earth is precisely 1,000 Cubits per second.

* This is the UK Nautical mile, rejected by the metricators.

** 100 fathoms = 1 cable; 10 cables = 1 nautical mile. -Ed.

What of a mathematical relationship between 18.24 and accepted constants?

A radian is the central angle inscribed in a circle and subtended by an arc equal in length to the radius of the circle. This angle is $57^{\circ}17'48.8''$. For most applications 57.3° is used. The related constant, Pi, was supposedly discovered by the Greeks only 2,000 years ago. However, Pi can be approximated by dividing one radian by the number of inches in a Cubit: 57.3 divided by $18.24 = 3.141...$ (and, of course, the number of inches in a Cubit can be found by dividing one radian by Pi: $57.3/3.141 = 18.24$).

Could 18.24 be as much a numerical constant as Pi? A definite relationship does exist between 18.24 and the 360° of a complete circle:

$360 / (18.24 \times 2) = 9.868$, whose square root is 3.141, Pi.

Another:

$18.24 \times 180 = 3,283$, whose square root is 57.3, a radian.

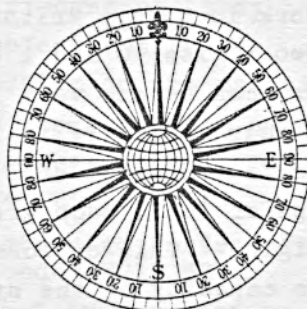
So, was the Cubit an ancient, accurate navigational unit?

* * *

Editor's Note

Mr. Corrigan's fascinating speculations continue from this point to develop an equally absorbing possible explanation of the Atlantis legend. The JOURNAL will publish a summary of that in a future issue if the curiosity of members has been aroused. Meanwhile, your reactions to this article are cordially invited.

* * * * *



SIXTEENMO ?

A BOOK was published (in 1862) in Philadelphia, advocating the use of sixteen as a radix - what is nowadays, with complete disregard for linguistic integrity, called 'hexadecimal' - with digits called:

An, De, Ti, Go, Su, By, Ra, Me, Ni, Ko, Hu, Vy, La, Po, Fy, Ton.

Also proposed was a year of sixteen months, with the names:

Anuary	Suvenary	Nictoary	Lamboary
Debrian	Bylian	Kolumbian	Polian
Timander	Ratamber	Husamber	Fylander
Gostus	Mesudius	Vyctorius	Tonborius

The author was, apparently, a successful engineer. Does anyone know his name, or what digital symbols he used? Some of these names are quite romantic-sounding and the repetition of endings in successive quarters is cleverly done.

Perhaps the computer people should be told.....Or perhaps not!

* * *

Sixteen, or onezen-four - 2^4 - is used as a counting-base in electronic computers and calculators, and it is the electronics industry which seems to have coined the term 'hexadecimal'. The digits, rendered in the seven-segment display, are:

1 2 3 4 5 6 7 8 9 A B C D E F 0

The logic 'gates' in these machines, of course, work in binary arithmetic, being essentially switches using two digits, 0 and 1 ('off' and 'on'). Binary numbers, however, can become very long and awkward to handle and store: this year in binary is 11111000110, for example. By using the fourth power of 2, long binary numbers can be encoded in groups of four, each group represented by a single 'hexadecimal' digit. This obviously saves storage space and increases the capacity of the system. The binary number given for this year will thus split into three groups

in the following way:

0111	1100	0110
7	C	6

-so this year would be stored as 7C6. Remembering that we are in base sixteen and that C stands for twelve, we can check that this is correct in both decimal and dozenal terms:

	DECIMS		DOZENS
7	7 x 256 = 1792	7 x 194 = 1054	
C	12 x 16 = 192	10 x 14 = 140	
6	6 x 1 = 6	6 x 1 = 6	
	<u>1990</u>	<u>1192</u>	

AURORA AUSTRALIS

A LIGHT HAS BEEN BURNING in the Antipodes. Fred Newhall, of the DSA, has sent literature of an Australian group with the purposeful name of The Modular Conversion Bureau. This has been functioning for some time and has not only declared for base twelve arithmetic (and measurements based on the foot) but has also come up with a complete new set of digital symbols and names. Here they are:

Y	U	T	K	V	H	S	O	N	X	E	F	Z
Yun	Dow	Tri	Kaw	Vin	Hex	Sef	Okt	Non	Xen	Elf	Zero	

The reason given for abandonment of the Arabic symbols is that of avoiding any possibility of confusion between systems of numeration. The name given to a dozen is 'Par', and number-names have been constructed up to Par Gross (the cube of twelve). Features particularly worthy of note are: (a) each digital name begins with a different letter (and for five that is V; for ten, X); (b) 'Elf', the German word for eleven, is used also by DSGB; (c) the symbol for four is identical to that used by Fritz Lang on his decimal clock in 'Metropolis'.

Well? What do you think?

Members' considered opinions are invited.

HOW METRICATION MAKES MEASURING MARVELLOUS

Q. Find the area of a sheet of building board, using (a) feet; (b) metres and (c) millimetres.

A. (a) Size = 8ft x 4ft, so area = $8 \times 4 = 32\text{ft}^2$

(b) Size = 2.4m x 1.2m. so area = $24 \times 12 \times 10^{-2} = 2.88\text{m}^2$.

(c) Size = 2400mm x 1200mm = $24 \times 12 \times 10^4 = 2,880,000\text{mm}^2$.

Now write a short paragraph explaining how you would convince an intelligent child that it is preferable to use the metric system for building work, giving reasons. Would the above metric examples be a useful way of teaching the concept of 'area' from actual practice?

* * *

The above is not a 'rigged' example: the sizes are those actually used in the two systems, for modular reasons. The method employed in schools to 'sell' the latter method(s) to pupils is simple and effective: they are neither taught, nor allowed to use, the former.

A DICTIONARY OF CANT

METRICATION. The most common argument for it is that it is desirable because it is inevitable, like Europeanisation. My old philosophy tutor would have called this a 'non sequitur', and my old driving instructor called it 'balls'. In fact, nature abhors metrication.

Nigel Burke

MATHEMATICAL SECTIONSUPERPRIMES

In JOURNAL 6, Spring 1198, Shaun Ferguson asked, among other things, whether anyone could find 'superprimes' (in any base) where, the prime being $abcd$, a , ab , abc and $abcd$ are all prime.

Charles Ashbacher, of the DSA, has found some answers.

Base ten

2333	2939	3739	7193
2339	3119	3793	7331
2393	3137	3797	7333
2399	3733	3739	7393

Shaun, however, admits that he omitted two conditions in the original question, which were: (i) there should be no repetition of digits in the number, i.e. $a \neq b \neq c \neq d$; and (ii) a , b , c and d are also prime digits (not including 1). Perhaps Mr. Ashbacher would be interested in this more difficult problem?

* * *

A second question asked: is it possible to find such a prime where any combination of the digits, taken one, two, three or all four at a time, is always a prime in a given base?

Charles Ashbacher points-out that, if we take a four-digit number such that

$$N = a.R^3 + b.R^2 + c.R + d,$$

where the base R is odd,

- (i) all digits may be odd, in which case N would be the sum of four odd numbers and hence even;
- (ii) one digit may be 2, which would make N odd, but here (with, say, $b = 2$) $b.R^2 + c.R + d$ would be the sum of one even and two odds, and so again even;
- (iii) similarly, with two twos (say, $c = 2$, $d = 2$) then $c.R + d$ would be even; and so concludes that there are no superprimes with an odd base.

* * *

A third question, noting that in base ten the number 19937 is prime even when 'circulated' (19937, 99371, 93719, 37199 and 71993 are all prime) asked if there were any similar examples in (i) base twelve or (ii) any other base.

A solution offered by William D. Nolte, Jr., observing that none of the digits can be even, is the result of a computer search of the combinations of odd

digits. In base twelve the only four-digit 'circulating prime' was found to be:

555£ 55£5 5£55 £555

and for five digits the trivial

11111

with no others in base twelve.

In base thirteen was found 35539;

In base fourteen none was found.

In base fifteen (and here it is necessary to change to the 'hexadecimal' numeration) were found:

1 7 A 1 C

7 7 7 7 C

7 A 7 7 A

In base sixteen:

1 5 9 3 A

3 3 1 3 C

1 9 7 7 7

3 5 5 A 5

3 3 9 1 1

1 A C 5 C

1 A C 5 1

1 1 1 7 3

1 1 3 3 9

as some examples. Charles Ansbacher found the solutions given in bases greater than twelve, and tells us that there are many more to be found in base sixteen.

PUZZLES

torn in two: (30) (25).

The number 3025 (base ten) is

Note that $(30 + 25)^2 = 55^2 = 3025$, the original number.

Another such number is 9801 (Check it!)

Assuming that in base twelve similar problems can be solved, are there four-figure numbers that will follow the same rule?

Some solutions: 3630 $36 + 30 = 66$ and $66^2 = 3630$;

££01 $££ + 01 = ££$ and $££^2 = ££01$.

Can readers discover some more?

What is the largest sum of money I can have without being able to give change for a sovereign? Solvers may use £sd (including farthings) or the Mickey Mouse money presently current.

There are *100 people employed on a short job. The total wages are *£100. The highly-skilled are paid *£10 each, the unskilled *£0'30 and child helpers *£0'15. How many workers are there in each category?

Set by Shaun Ferguson (after Dudeney).

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will help!). Back numbers of DOZENAL JOURNAL
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