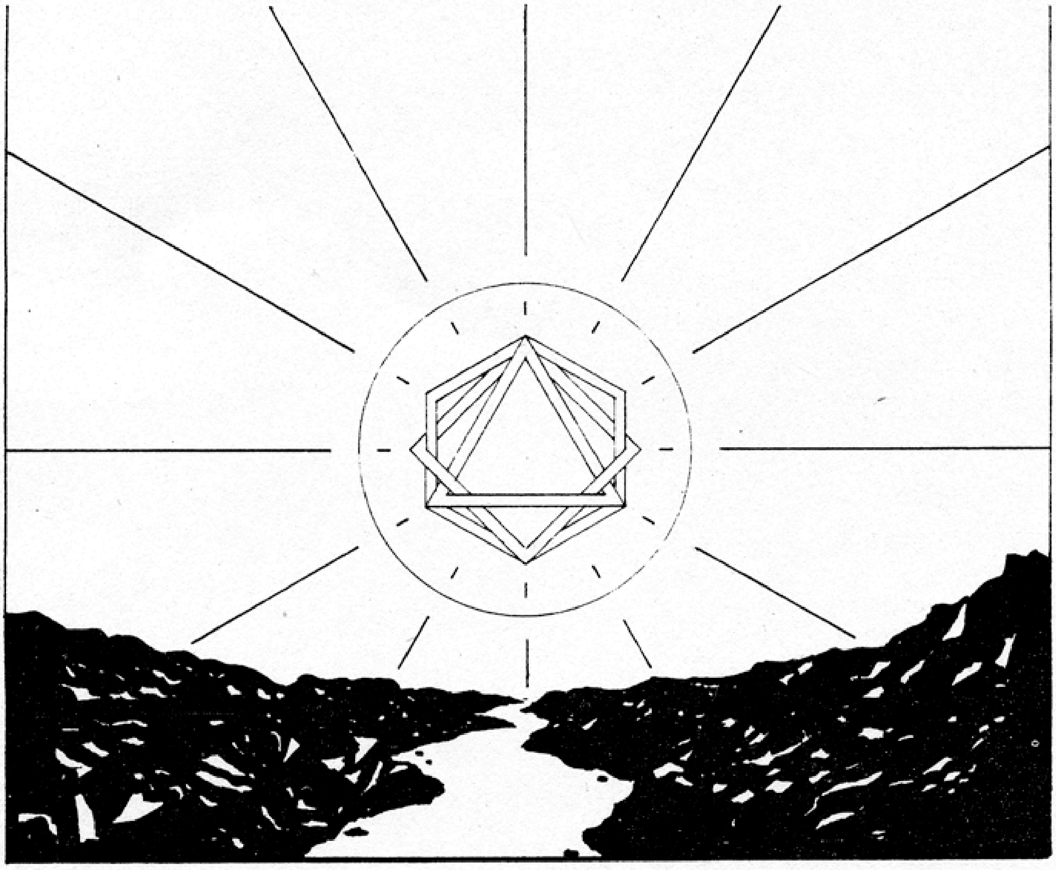
THE DOZENAL JOURNAL



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THE DOZENAL JOURNAL

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"The requirements of ordinary life in Babylonia, as everywhere else, demanded the use of the common fractions of a measure, in this case ½, ⅓ and ⅔."
Karl Menninger,
NUMBER WORDS & NUMBER SYMBOLS

In this Internet version of the Dozenal Journal no. Ɛ, dozenal numbers are marked, where not otherwise indicated, with an asterisk (*). The 'fractional point' is the apostrophe (e.g. 0·5 = 0'6). The standard DSGB symbols for ten and eleven (Ɛ and Ƹ) are used. Don Hammond's version of Ɛ was used throughout the original printed version of the Journal.

EDITORIAL

JACOBINS OR JACKASSES?

Both, I think. In celebration of their bicentennial year, we have the New Jacobins of the EC - the boys from Brussels - on a joyous victory rampage of kicking to bits what is left of Imperial/rational measure and generally showing the Brits who's Boss Around Here. They have learned that it is not necessary - indeed, is counter-productive - to subjugate people by the messy business of cutting-off heads; all that is needed is to subvert and then destroy their culture.

Initially, of course, it can be desirable to strike a quick, decisive blow which will inflict strategic damage before the victims have time to realize what is happening. With Pizarro and his Spaniards in Peru it was the deceitful capture and subsequent execution of Atahualpa, their king (whom they had supposed to be invulnerable), which demoralized the Inca and led to their enslavement. For the British, the killing of £sd was the essential breaching of defences: the disarming of Britannia had to precede the cultural rape now nearing completion (it is insolently symbolized in the coinage: compare an old penny - Britannia sits, dignified and upright, holding her trident erect - with the decimal 50p piece, where the trident is laid back compliantly, the shield is pushed aside and the lady, now clad in diaphanous garments, displays herself looking more like a complaisant odalisque than a guardian).

We have our *own* Jacobins (people we might once have called quislings) who, avid for Napoleonic Europe, have infiltrated Government and Civil Service alike and have, since the late 1950s, lost no opportunity to denigrate and ridicule everything British. Institutions and customs were castigated, Imperial overseas responsibilities shed. (Imperial uniforms were sold in London as fancy dress for the permissive-society mayflies of the pop world) and the people generally 'softened up' in readiness for the loss of their heritage.

The success of these saboteurs, however, was - is- dependent upon the Jackasses: those people of flawed education, limited perception and a slavish obedience to fashion who were - are - in positions of influence. Many MPs, teachers, managers, trade-unionists, etc., actually came to believe Europe to be Better In Every Way and so joined enthusiastically in the great surrender which began with that decisive blow in February 1971 and is now almost finished.

Weight of culture

SIR—*Nature* suffers from tunnel vision in smugly condemning Americans for preferring the British system of weights and measures (*Nature* **344**, 575; 1990). This attitude fails to appreciate that units of measurement are not merely calculating devices, but integral components of a nation's cultural matrix. As such they are the numerical equivalents of the languages, traditions and customs that identify and enrich us both collectively and as individuals.

The ineluctable march of the metric system represents the victory of cold calculation over the ebullience of the human spirit. Once its triumph is complete, the world's cultural gene pool will have become further depleted and humanity reduced one more step towards the mentality of the average robot.

C.H. EVANS

Blackheath Grove,
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NATURE · VOL 345 · 21 JUNE 1990

Virtually every intelligent human activity is permeated by measurement. What we see now - and what will continue unless a very unlikely reaction sets in - is a mopping-up operation as Brussels issues directive after directive, with no resistance from a nation whose freedom has been signed away. All remaining pockets of rational measure are to be inoculated with the culture-killing metric virus until only miles on our roads and pints in our pubs remain to suggest that this was once Great Britain - and how long will the last as we allow ourselves to become, like Rossum's Universal Robots or, perhaps, like the Eloi in Wells' "Time Machine": a domesticated herd of near-identical and programmed consumer units, devoid of initiative and numbed from development by the anaesthetic decimal?

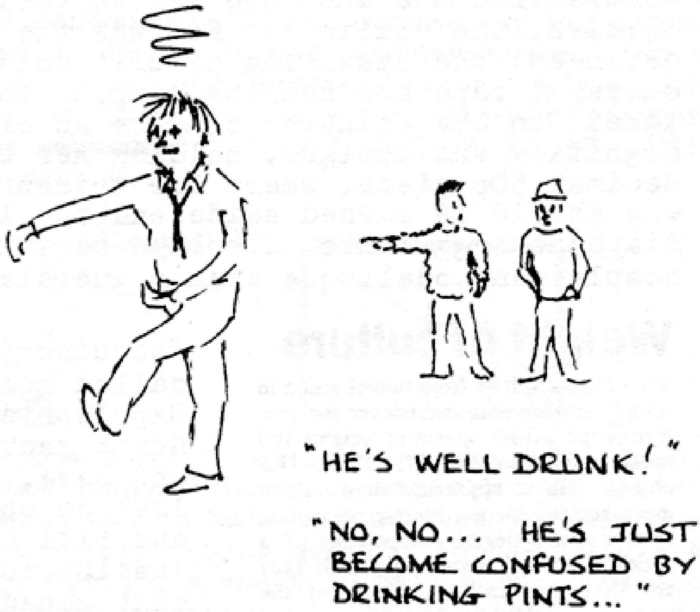
THE PINT IS TOO CONFUSING BY MILES

THE pint and the mile should be abolished when Britain goes metric, says a leading consumer body.

Serving beer by the pint while other drinks are bottled in metric units will only confuse customers, according to the National Federation of Consumer Groups.

Common Market chiefs have promised a permanent exemption of the pint for serving draught beer and an indefinite exemption for the mile when Britain is brought into line with the rest of Europe.

The federation wants 1999 set as the deadline for abolition of the mile.



Europe = metric = decimal. We have had a general election in which the choice lay among pro-Europe Conservatives, even-more-pro-Europe Labour and fanatically-pro-Europe Liberal Democrats. Some of us may feel disenfranchized. Should we go on? Should we continue the pursuit of dozenal arithmetic and rational measure in the face of implacable hostility? I think so; but we need encouragement: articles, letters (both to the Journal and to the Press, etc.) and ideas. Let us hear from you!

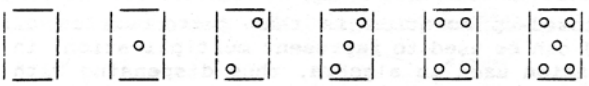
The Editor.

Counting In Sixes

by George Jelliss

The arabic numerals 0 1 2 3 4 5 are adequate for positional numeration in base six, however I have investigated other notations, with the aim of finding one that is not merely an arbitrary choice of symbols.

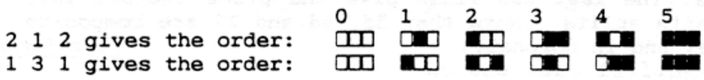
The Six Faces of a Die. One such system is found in the design of the faces of dice (replacing the six by a blank as on dominoes):



These arrangements of dots are the most symmetrical possible (so that they can be easily recognised from which ever angle the die falls) and the most compact possible (so that a good size dot can be used for clear visibility) within the square area. The other choices, i.e. two or three dots in line parallel to a side, or four dots one in the middle of each side, are more cramped. Also three dots in a triangle do not match the symmetry of the square. These patterns are the optimal solution to the problem.

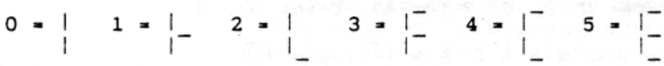
Counting Sticks. Another approach was to consider representing each digit by a pattern of black and white areas. The simplest case would be a row of three areas: □□□. These can be coloured black or white in 2 (i.e. 8) ways. But if we regard the three areas together as like a stick that can be turned end for end then we have only six distinct cases: □□□, □■□, ■□□ (= □□■), ■□■, ■■□ (= □■■), ■■■.

In what order should these symbols be arranged to represent the digits 0, 1, 2, 3, 4, 5? Counting the black areas (i.e. representing black marks by 1s) gives a partial ordering, with □□□ first, then the two with one black mark, then the two with two black marks, and ■■■ last. Then I thought of giving the end positions different values from the one in the centre, chosen so that ■■■ totals 5: two valuations are possible: 2 1 2 and 1 3 1, and each determines a unique sequence!



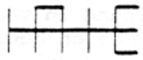
The first solution is best as it conforms to the partial sequence given by counting black marks as ones.

A Segment Notation for Six Digits. Applying this result to the seven segment display we note that there are three horizontal bars that can be on or off. Thus we can use these to represent the segments of our counting sticks. To connect up the horizontal bars, and provide a symbol for zero, we can insert two vertical bars, giving the notation:



It will now be apparent why I chose to place the vertical lines on the left and to choose the bottom bar for the 2 and the upper bar for the 3: the symbols resemble the letters I, T, L, F, C, E, except that the T is on its side and the C is rather rectangular.

Writing and Printing in Hex Notation. In writing the digits by hand I find two further conventions helpful. First I join up the digits in a number to form a single hieroglyph, and since this would cause zeros to vanish, I insert a diagonal ligature after each zero (thus converting the I into a V symbol - V for void perhaps). For example:

119↋' = 1991· =  = TFTTE 1000' = 1728· =  = TLVVV

An advantage of this closed-up notation is that juxtaposition of numbers (without joining) can be used to represent multiplication, in concordance with the notation used in algebra, thus dispensing with the need for a multiplication sign. For printing, I have been able to make up a set of symbols based on the "Swiss" sans serif style as follows: **V, ⊢, L, F, C, E (e.g. the C is L overprinted with greek Γ).**

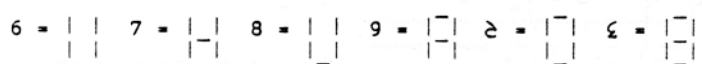
Table of Numbers in Hex Notation. The following table shows all hex numbers of 1 or 2 digits.

Dozenal:	0	1	2	3	4	5	6	7	8	9	↊	↋
Decimal:	0	1	2	3	4	5	6	7	8	9	10·	11·
Hex:	V	⊢	L	F	C	E	⊢V	⊢⊢	⊢L	⊢F	⊢C	⊢E
Dozenal:	10'	11'	12'	13'	14'	15'	16'	17'	18'	19'	1↊'	1↋'
Decimal:	12·	13·	14·	15·	16·	17·	18·	19·	20·	21·	22·	23·
Hex:	LV	L⊢	LL	LF	LC	LE	FV	F⊢	FL	FF	FC	FE
Dozenal:	20'	21'	22'	23'	24'	25'	26'	27'	28'	29'	2↊'	2↋'
Decimal:	24·	25·	26·	27·	28·	29·	30·	31·	32·	33·	34·	35·
Hex:	CV	C⊢	CL	CF	CC	CE	EV	E⊢	EL	EF	EC	EE

Primes and Primals in Hex Notation. As Donald Hammond has noted in DJ#5 page *30 all prime numbers greater than 3 are of the form 6n±1. I call these numbers primal. In Hex a number is primal if and only if its units digit is 1 or 5. The primals are the primes greater than 3 and the products of such primes, or in other words numbers not divisible by 2 or 3. The following is a list of the first few composite prinals. The last two lines give the prime factors into which each composite splits. (Note that 55, 65 and 77 are composite primals in decimal and in dozenal!)

Dozenal:	21'	2↋'	41'	47'	55'	65'	71'	77'	7↋'	97'	↊1'	↊5'
Decimal:	25·	35·	49·	55·	65·	77·	85·	91·	95·	115·	121·	125·
Hex:	C⊢	EE	⊢L⊢	⊢F⊢	⊢CE	LVE	LL⊢	LF⊢	LFE	F⊢⊢	FL⊢	FLE
Factors:	E^{L}	E	$⊢⊢^{L}$	E	E	⊢⊢	E	⊢⊢	E	E	$⊢E^{L}$	E^{F}
		⊢⊢		⊢E	L⊢	⊢E	LE	L⊢	F⊢	FE		

A Segment Notation for Twelve Digits. This hex notation will give a dozena]. notation by using the other pair of vertical bars to add 6. The resulting shapes are also dislexia-proof and also translate into cognate capital letters: N, H, U, R (or A), D (or 0), B.



In writing, the two bars for 6 can be distinguished from two zeros by joining them with a diagonal. The fact that I represents zero and 0 represents ten may take getting used to! But that's how it works out.

ITEMS

ELECTION NOTE

In the 1992 Election Manifesto of the Monster Raving Loony Party, whose leader is Screaming Lord Sutch, one of the principal objectives was the decimalization of time. That seems to put the idea in just about the correct perspective..

SPACED OUT?

On a recent Scale of Charges for private use of official telephones, Hampshire County Council gives the radius outside which charges increase abruptly to 'long distance', or 'trunk' rates as 56·4 kM. Now, *that's* got a ring to it, has it not? How much more impressive it is to say: "Fifty-six-point-four kilometres" than to mumble on about old-fashioned stuff like "thirty-five miles'.

Makes one really believe in progress, doesn't it?

NEEDS MUST WHEN THE DEVIL DRIVES.

To combat the rising incidence of serious road accidents involving young, inexperienced drivers, the Automobile Association has proposed that an automatic period of disqualification should follow accumulation of six penalty-points during the first year after passing the test. The limit would be raised to *nine* points during the second year and to the *usual twelve* points thereafter.

A clear example of sensible, duodecimal thinking? Certainly, the intermediate nine-points limit would be impossible with decimals: there is no such thing as half a penalty-point.

THE BAD OLD DAYS

Back in pre-decimal times, British people carried their weights and measures in their pockets and purses. *Three pennies (3d.) weighed one ounce* as did five halfpennies or ten farthings. Four shillings' worth of pennies made 1 lb.

The halfpenny was exactly one inch in diameter and the penny, one-and-one-fifth inch; thus, twelve halfpennies side-by-side measured one foot, as did ten pennies. The sixpence was exactly three-quarters of an inch across, so two sixpences gave 1½ inches and sixteen of them made one foot.

Thus, one could make quick checks of weight and length with one's small change. No doubt we should all be grateful that decimalization has rescued us from such dangerous, old-fashioned nonsense.

Andrew Alexander

SOMEHOW, I missed the bonfires and the dancing in the streets. All the same, I am assured that the Royal Mint invited us a few days ago to celebrate 21 years of decimalisation.

The Mint apparently thinks 21 still marks a coming of age. It would be nice to think that somewhere in that organisation there is still a hankering after guineas. Decimalisation can now be ranked, with so much else, as one of the disasters of the Sixties and Seventies. (Please do not write in saying it caused inflation too — it didn't).

The excuse for decimalisation was simplicity. But simple systems breed simple minds. And the inability of so many younger people today to cope with figures of any complexity owes a lot to decimalisation.

A great advantage of the old and 'difficult' system was that children learnt it quickly. Money, as ever, mattered. Multiplying and dividing by 12 and 20, knowing how many threepenny bits there were in a shilling, how many in a florin and how many half-crowns made a pound became routine.

There is always something to be said for the difficult. I have long suspected that one reason for the apparent high intelligence of the Japanese is the complexity of their vocabulary and its characters. Children have to master it from an early age and mental agility is automatically stimulated.

As if decimalisation was not bad enough — and only some tourists actually seemed stumped by the old system — we added the abomination of metrication.

Those who believe that metrication, being simple, is in some way 'natural' overlook the sheer unnaturalness of that system. Inches, feet and yards are based on human measurements. By contrast the metre was supposed to be 1/10,000,000 of a quadrant of a circle of the Earth measured around the poles of the meridian passing through Paris.

It would be difficult to imagine a more grotesque basis for daily mensuration. The metre has no human dimensions at all. No man has such a stride. Nor has the centimetre any relationship to the human hand. An acre is also a homely measure. The hectare is far too large.

One grumble about the kilometre: Britain has been 'allowed' by Brussels — such a kindness, don't you think? — to keep the mile. Yet news reports constantly refer to distances in kilometres.

The reason may well be that, where the original distance was given in kilometres, the reporter is incapable of turning it into miles. That simply bears out my point about the collapse in numeracy and mental agility.

The manifestation of this is particularly pathetic in shops. Young Noyleen stands at her cash register, slowly ringing up 25p and 35p and 10p — and is awed to find that old codgers like me have already put down 70p and are getting impatient.

Calculators should also be rationed for the young. Apart from eliminating needed mental exercise, they rob people of a sense of size. Results ten or 100 times too large or too small are not immediately seen as obviously wrong.

Even business graduates are affected. 'That's about 1,000 square feet', I told one when we were looking at a space about 18 feet by 52 feet. 'How did you know?' he asked after fiddling with his calculator. Oh dear.

Daily Mail, Friday, February 21, 1992

RATIONALITY

Troy

TERMS must be defined. In doing so here, I realize that much of what I write will be well-known to many members - though not all - but it does no harm to confirm these matters.

The word 'rational' is directly related to 'rate' and. 'ratio', to 'ratify' and 'ratiocination'. It implies the use of reasoning, of proportion, of due measure; it suggests pragmatism and common sense. A mathematician will call common fractions 'rational numbers' because they are *ratios* and so can be used for practical calculations involving only the simple arithmetic which is (or should be) available to most ordinary people in their daily affairs.

Rationals are 'sensible' numbers in the same way, perhaps, that *heat* which causes a change of temperature is called, by engineers and physicists, 'sensible heat'. The other kind of heat - latent or 'hidden' heat causes a change of state (if you continue to heat boiling water it doesn't get any hotter but instead turns to steam) but no change of temperature; the only way we can *measure* latent heat is to convert it to the sensible variety. In the same way, *irrational* or 'hidden' numbers, like π or $\sqrt{2}$, need to be measured by conversion into rational approximations if they are to be used for practical purposes.

There was a somewhat far-fetched story in which a factory work-robot was instructed to electroplate a copper disc with platinum. Before issuing an ingot of platinum to form the anode, the stores department insisted on an exact specification for the amount of precious metal to be used. After some days had passed with no sign of the work an enquiring manager discovered the hapless robot trying to find an *exact* value for π; it had not been told to use a rational approximation Again, on photocopying machines the enlargement ratio from A4 to A3 is given as '141%', never as the $\sqrt{2}$ it is supposed to be.

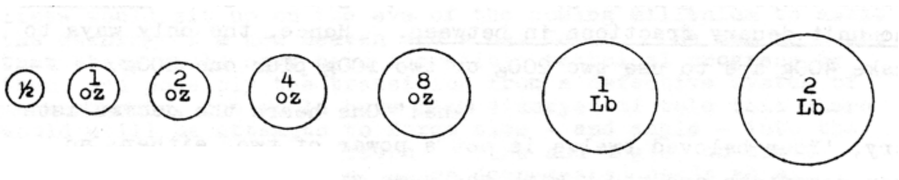
So, rational numbers are the ONLY kind we can use for measurement. It follows, surely, that measurement *systems* should accommodate, as fully as they can, those rational numbers - ratios - which arise naturally from basic, everyday considerations of geometry, proportion and practical economy. In this context, American author Donald Kingsbury once observed that traditions are 'solutions for which we have forgotten the problems', with the corollary that discarding the traditions without due thought brings the problems back again .... In this article I shall look at examples of traditional measuring systems and how the problems they solved are now returning to bedevil us as decimalization wreaks its damage; and suggest how the Rational approach can both preserve and enhance these hard-won and - yes! - advanced principles which our political masters would like us to forget.

All approximations are, speaking mathematically, rational numbers; but they are not always very *sensible* numbers to use if simpler ones can be chosen. While no-one can avoid approximations for irrational numbers, it *is* often possible to eliminate nuisances like 0·166... and shorten such as 0·4375 by using numbering and/or measuring methods more suited to the work and ratios demanded. I use the term 'Rational' - with capital R - to denote scales or units which are not only ratio-based but are also sensible (i.e. as simple as possible). It is this principle, this mode of thought, which I call 'rationality'.

HOW TO LOSE WEIGHTS

Mathematics tells us that, in many cases, binary is best. The powers of two constitute the ‘minimum’ power-series for weights on a two-pan balance (with weights one side and goods the other) since it specifies the least number of different weights needed to cover all numbers of units. The rule about weights is that the subsidiary pieces should be simple unit fractions whose denominators are factors of the basic standard. Hence, for example, including ¼ lb. and ½ lb. pieces makes a ¾ lb. piece unnecessary.

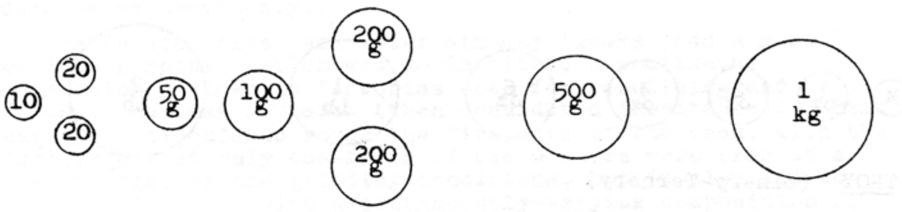
Here is a 7-piece set of English kitchen-weights (still much used in English kitchens) of the system dating from e. 1290 (*8£6).



AVOIRDUPOIS (Binary)

All intermediate weights, in ½-oz steps, can be made from combinations of these pieces, so the set requires only one of each. Use of this system is now effectively forbidden in British schools.

By comparison, the decimal metric set of weights needs to *duplicate* some:



METRIC (Denary)

Thus, the binary set, needing only three pieces for every four in metric, is more efficient in use and cheaper to make than is the denary set.

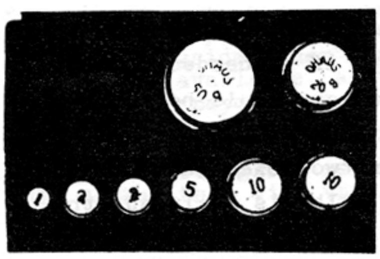
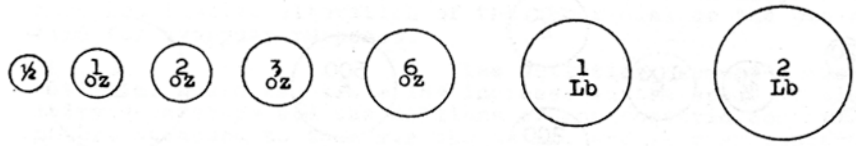


Photo at left is of a set of ‘Student’ laboratory weights. In this case, the 2g and 10g are duplicated. This is cheaper than having two 20g, but the need for duplicates remains.

We can see the basic flaw in the decimal arrangement if we remember that subsidiary weights have to be *unit* fractions of the standard, so the next larger piece after 200g (1/5 kg) has to be 500g (1/2 kg). *There are no unit denary fractions in between.* Hence, the only ways to make 400g are to use two 200g or two 100g plus one 200g.

'Aha!' One hears the decimalists cry, 'Your beloved twelve is not a power of two, either; so you dozenists are stuck with the same problem.'

Well, not quite. Twelve accepts 2 and 4 as factors; it accepts also 3 and 6. Noting that 4 is *twice* 2 and that 6 is *twice* 3, we see that it is possible to use a binary multiplier (and so retain binary efficiency) and simultaneously introduce the second prime, 3, to design a set of weights for a dozenal system. Let us now incorporate this scheme into something like the Troy pound of twelve ounces (even older than Avoirdupois):



TROY (Binary-Ternary)

Just as with pure binary, all intermediate weights can be achieved by combining others, so we need only one of each size.

There is more. It will not have gone unobserved that 3oz, 6oz and 1 lb. can be made from combinations of lower values; in fact, if we needed to go only as far as a dozen ounces, the 1 lb. weight would be superfluous. Including the 1 lb., therefore, allows further weighing up to and including 2 lb. or two dozen ounces *without the need for a 2 lb. piece*. If the 2 lb. is included, the range extends to 4 lb. inclusive

The binary (Avoir.) and dozenal (Troy) sets are easy to use and need only seven weights each.

The dozenal set has an added advantage in that it can give the full 4 lb. while the binary misses by ½ oz. The decimal set is not quite so easy to use and - more seriously - involves nine weights rather than seven and is thus more bulky to store and uses more metal in manufacture.

Note also that with a twelve-ounce pound so divided, a more flexible range of fractions is available, including thirds and sixths as well as halves, quarters and eighths.

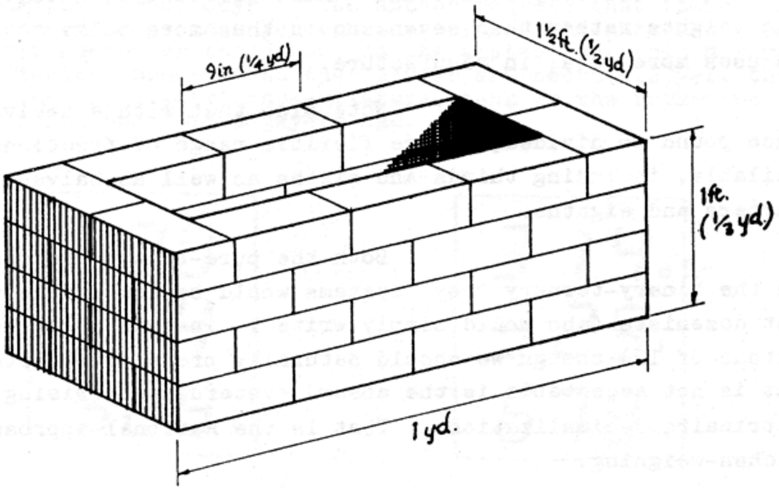
Both the pure-binary Avoirdupois and the binary-ternary Troy systems would be acceptable to most dozenists (who would simply write 14 instead of 16, or 10 instead of 12) though we should naturally prefer the latter. What is not acceptable is the absurd wastefulness arising from doctrinaire decimalization. That is the Rational approach to kitchen-weighing.

BRICKBATS

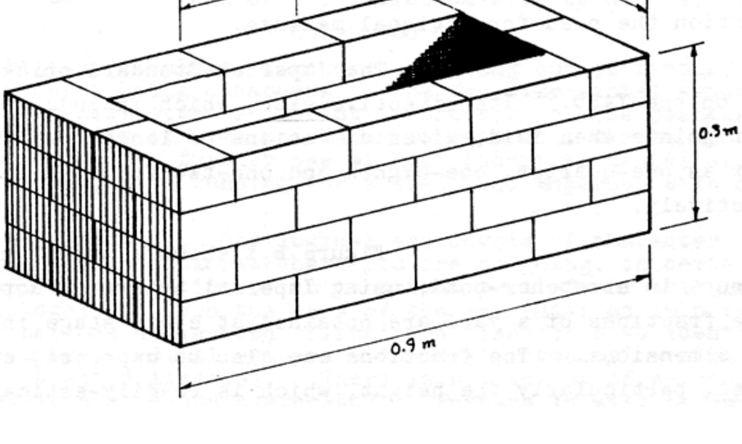
The housebrick has been mentioned before in the JOURNAL (see No. 3), but deserves a closer look; embodying as it does a solid, three-dimensional actuality, this humble yet essential artifact illustrates to perfection the need for Rational measure.

The Imperial Standard brick is based on the YARD. Its effective size, which includes the mortar joints when laid, gives dimensions of length, width and height as one-quarter, one-eighth and one-twelfth of a yard respectively.

Figure B.1 shows a modest brick structure in stretcher-bond, using Imperial bricks. Note how simple fractions of a yard are obtained at every stage in three dimensions. The fractions can also be expressed easily in feet, particularly the height, which is readily-estimated on site at four courses to the foot. A builder told that a wall rises n feet from the DPC knows that 4n courses of bricks will be needed; a similar simplicity obtaining for horizontal dimensions gives a whole number of yards or feet for every four bricks in stretcher bond.



B.1



B.2

A glance at figure B.2, which is the same structure made from 'metric' bricks reveals that these will not fit a *metre*, either lengthwise or coursewise.

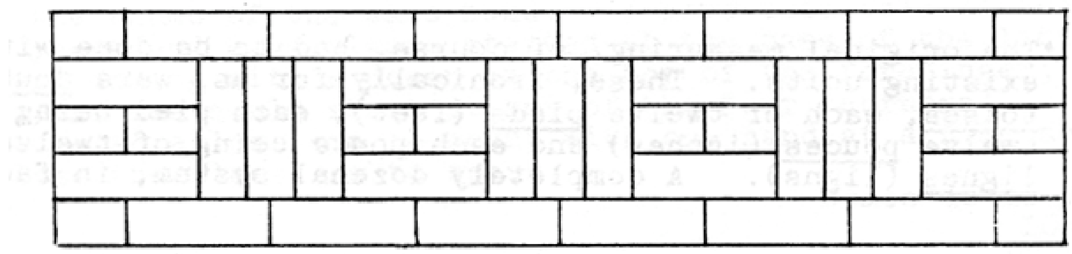
The desirable ratio - in lowest terms - of brick dimensions is 6 : 3 : 2,.and this ratio cannot be obtained with metric units (try it!); the pathetic result, therefore, of the socalled ‘metrication’ process in the building industry, which was undertaken for political, not ergonomic, reasons has been the invention of the ‘metric inch’ of 25mm, the ‘metric foot’ of 300mm and the ‘metric yard’ of 900mm (not that anyone is officially allowed to say so). The ‘metric standard’ brick, laid in mortar, is thus given dimensions of: thickness 75mm (3 metric inches), width 112·5mm (4½ metric inches) and length 225mm (9 metric inches); thus sized, these bricks can be laid four courses to a metric foot and four lengths to a metric yard.

Hence, the price paid for ‘metricating’ the housebrick is abandonment of the metre itself: the *primary* unit, the Emperor of the metric system in his grand decimal raiment, has arrived at the builder’s Yard and tripped over a brick ....

(No; this is not just a British reaction: the French themselves do not use the metre as a building module.)

This ‘metric’ brick is very close in actual size to the Imperial. It is a little smaller and will lay to the yard and foot; so if you want to lay bricks stay with your folding yard and avoid wasting money on a folding metre that will not fit the work. (What a spiteful little change this is!).

Again we see that the criterion for efficient measuring units is the ready accommodation of ratios suitable for the work. The fabric of reality is tough and trying to cut patterns in it with blunt decimal tools is a self-defeating exercise.



CHOOSING THE RIGHT ANGLE

The metre has been referred-to as the ‘Emperor’ of the metric system, which it is; but Emperors do not spring from nowhere: they result from some or other method of selection. Most who have taken any interest in these matters know that the metre is - or was originally supposed to be - one ten-millionth of a quadrant of the Earth from Pole to Equator.

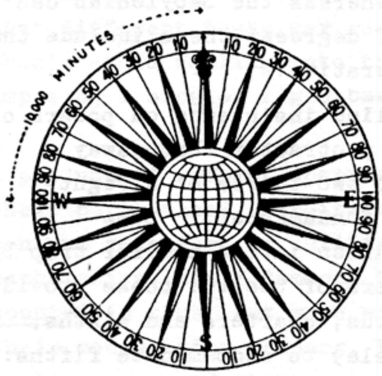
So, in an act of breathtaking contrariness, the very first and fundamental decimal-metric operation was the *denary subdivision of the quadrant.*

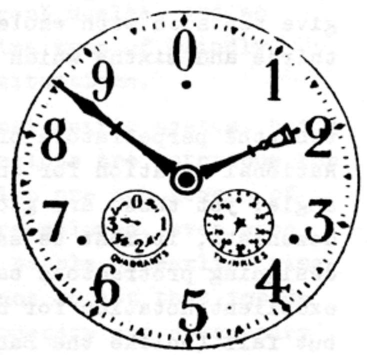
Now, numerous proposals have been made - by dozenists and others - regarding angular scales: most of these have been based on the circle or half-circle. Yet, as the French saw clearly, it is the right-angle, or quadrant, which really matters, for that is literally the corner of the three-dimensional world; and they divided the right-angle into one hundred Grades as the basis of a decimalized protractor. The length of arc at sea-level which subtends an angle of 1 Grade at the centre of the Earth was then found by direct measurement. This distance was divided by one hundred thousand to give the metre.

[The original measuring, of course, had to be done with existing units. These, ironically for us, were *double-toises* each of twelve *pieds* (feet), each pied being of twelve *pouces* (inches) and each pouce being of twelve *lignes* (ligns)

A completely dozenal system, in fact ...]

The kilometre was - and is - seen as a navigational unit: one hundred kilometres along a Great Circle is equivalent to one Grade on the denary protractor. Navigation, however, is not only a matter of angle, but also of time; the decimal clock is a necessary adjunct to the Grade protractor. The diagrams below were both taken from an article published in 1906 (*112↊), strongly advocating the system.





It should be noted that children in State schools in Britain are being taught elementary navigational mathematics *exclusively* in terms of kilometres. Decimal-clock suggestions keep popping out of the woodwork and at least one Town Council (Leeds) has switched to decimalized time-sheets for its staff.

The Babylonians, developing Sumerian concepts, used sixty as a secondary counting-base. It seems probable that they arrived at the protractor we still use today by taking the natural *sextant* (one-sixth of a circle obtained by stepping-out the circumference with its own radius) and dividing it into sixty degrees. This automatically conferred ninety degrees on the right-angle: a very good number which caters well for the prime constructible angular divisions of the circle (halves, thirds and fifths). The scale itself, however, cannot be constructed in the plane and needs three-dimensional manufacturing methods.

By contrast, the Grade scale also inconstructible in the plane - cannot accommodate thirds; under its regime the draughtsman's familiar and indispensable 'thirty-sixty' set-square, giving one-third and two-thirds of a right-angle, would have to become 'thirty-three point three recurring/sixty-six point six recurring' set-square. As Oliver Hardy would have said: 'Another fine mess!'

The centesimal Grade protractor can manage only six exact subdivisions of the right-angle if whole numbers of grades are used, whereas the Babylonian can give ten such with whole numbers of degrees; these include the thirds and sixths which are so imperative.

By blind insistence on powers of ten, the perpetrators of the Grade protractor threw away Rational notation for one-third and two-thirds of a right-angle; yet these are geometrically fundamental. Some dozenists, it must be said, have fallen into a similar trap by designing protractors based on powers of twelve: these provide excellent notation for halves, thirds, quarters and sixths, but fail (unlike the Babylonian scale) to accommodate fifths. As was explained at some length in an earlier article (JOURNAL 8, p.*11), five, while not important as a linear division, is significant in angular measure; hence, the Babylonian device is of better rationality than either pure decimal or pure dozenal versions.

The Babylonian protractor can be improved: the Rational protractor (REVIEW No. *30. p.↊) applies the classic sexagesimal scale to the *quadrant* instead of the sextant: this gives a scale of sixty or five dozen Rates (ø) to the right-angle (thus allowing both decimal and dozenal notations to be numbered roundly), accommodates 2, 3 and 5 as factors and can be constructed in the plane. Both Babylonian and Rational protractors fit the existing clock; the Grade protractor does not, of course.

PROTRACTOR	DIVISION OF QUADRANT	PRIME FACTORS 2	3	5	COMPATIBLE WITH CLOCK	CONSTRUCTIBLE IN THE PLANE
Babylonian	•76 ⌿ 90	●	●	●	●	
Grade	•84 ⌿100	●		●		
TGM	•60 ⌿ 72	●	●		●	
Rational	•50 ⌿ 60	●	●	●	●	●

MEASURE FOR MEASURE

The lesson we should draw from these observations is surely one of disciplined flexibility: *we must recognize natural constraints and patterns,* simple proportions and efficient styles of measurement. We can see that different tasks may require different scales, and so should avoid falling into the doctrinaire trap of blindly imposing a single, rigid basis to all situations.

The decimal-metric system, being a product of revolutionary zeal (and zealots are notorious for their puritanism), permits no units which are not powers of ten: no secondary or auxiliary bases are allowed, even when mathematics itself demands them. The people of earlier times counted in tens, but were wise enough not to let that impede their mensuration: binary, ternary, duodenary and sexagenary scales were used where appropriate; no-one felt threatened by them. It was realized that powers of ten, though perhaps good enough for mere counting, raised unnecessary barriers to sensible working practices; and so such numbers were largely rejected for units of measurement. Decimal *currency*, even, was abandoned c.130 BC.

[The denarius, as its name suggests, was originally ten As, but was made worth sixteen As at this time. Some assert that this was merely devaluation of the As; but in that case why choose sixteen?]

There is another irony here: because our forebears (not frightened of fractions) were happy with 8-pint gallons, 3-foot yards and so on, they were free of the stifling influence of the denary base (used solely for simple arithmetic) and so did not bother about changing it; *decimal numeration survived by being marginalized* Had there been some sort of cosmic law which ordained a match between number-base and measures, we should have had a twelve-based numeration from time immemorial (especially once it was found that it made calculations easier, too!).

Yet... We all recognize the convenience afforded, particularly to the scientific world, by measuring-units which fit the number-base: a match between the two schemes, whereby successive units of measure correspond to successive powers of the radix, so permitting standard-form calculations and fraction-point transformations, is highly desirable to laboratory workers and accountants alike. It promises coherent systems and hence elimination of troublesome conversion-factors. It was this promise which seduced - and still seduces - academics and politicians (for different reasons) into uncritical acceptance of the decimal-metric idea.

They have been sold a pup. What looks so good on paper, with its elegant unit names and inspired series of power-prefixes, fails to accommodate natural ratios, often imposes problems where there were none before and has a marked propensity for expanding simple fractions into strings of decimal digits. Instead of grasping the nettle of decimal incompatibility with natural mensuration and arithmetic, L'Institut National shrank away from the chance of basing their system on the dozen and went for a quick denary fix.

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Our dozenal base *is* amenable to true rationality: we have seen how a twelve-based weight system equals and sometimes betters the binary; how linear, areal and cubic measure, using feet-and-inches and the almost miraculous yard, are elegantly served. Accepting secondary bases where appropriate (so avoiding the disastrous rigidity of the metric system) we can have, for example, our inches divided-down dozenally in a power-of-twelve system, yet leave the other edge of the rule with the binary subdivisions which are so useful; we can have an even better protractor than we have now; we can leave the clock-dial alone (apart from the renumbering it always needed anyway); we can have a thermometric scale from 0 (freezing water) to *130 (boiling water) using Fahrenheit degrees; and - underlying it all - we can have the most efficient and (if I may use the expression) user-friendly arithmetic it is possible to devise.

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I have two comments on matters in the latest issue (J9). First, 'Dial-A-Chord' looks like a good device for elementary music students. G minor is indeed a reflection of D major if we recall that the minor triad may be formed by taking the undertones 4-5-6 of a pitch, whilst the major triad results from the same overtones. The undertones of D, therefore, give the G minor triad. This is not necessarily the true origin of the minor triad: that is a matter of some dispute in music theory.

My own work involves tuning systems of every imaginable kind except the common one so well illustrated in the Journal. Whereas I am an ardent dozenalist in nearly everything, I avoid twelve completely in my research and most of my compositional

As for the clock given on p.*14 (Charles Field's CMS Clock), my opinion has always been that to maintain two cycles in a day (of hours or whatever they may be called), or to have a hand rotate to *20 of something, is unnecessary. I much prefer having nested dozens in a day, so that each hand moves from 0 to 1 or *10 or *100 in one cycle. On a digital clock, four numerals related in this way may specify time at a glance to the nearest 1/10000 day. This is in fact the basis of the model I had built a few years ago and which is running as I type this letter. If we keep the double cycle in a day, we may just as well keep 16 ounces in a pound, and so on.

And with all due respect to those who want a return to £sd, I find the use of anything but nested dozens in monetary units to be pointless. Bring on the pound of *100 pence, provided we have coins of *60p, *30p, *10p, 6p and perhaps 3p and lp. (I realize the last are not worth much, even less when there are *100p per pound instead of 100) The North American, Southern Pacific, etc., dollars should obviously be similarly divided, bringing to an end the validity of the well-known saying: "As phony as a three-dollar bill".

Paul Rapoport

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Various mentions of five-finger counting in the last six issues of the JOURNAL seem to suggest that God got it wrong, or was sabotaged, when making Man who arrived with five fingers. But God got it excellently right: it was Man that interpreted it wrong.

One should not count individual fingers of one hand with the opposite hand; one should count the segments of each finger with the thumb of the same hand. I have used this method for many years and have always started at the base of the little finger, ending with a dozen at the top of the forefinger; but I suppose one could start at any of the four 'corners'. This method also allows one to count with one hand at a time and not lose the place, so to speak.

James de la Mare

LONDON

Mr. Robin Hancock has sent us a tape on which he expresses his view that decimalization/metrication is for cultural, not mathematical, reasons. In fact, he says, reasons are not given: the attitude is "accept it or else...". Mr. Hancock tells us that he met the metric system at school and even then reacted instinctively against its rigid denary inflexibility, its "user-unfriendliness". He feels that a whole generation is being indoctrinated as a result of a weakness in the national character (and is not sure whether this weakness is inherent or instilled).

Mr. Hancock also points-out that the English-speaking peoples gave rise to the most successful culture ever, and it is that which is under attack: the greatness of Britain, he avers, owed much to her *breaking-away* from Europe and forming a muth better society subsequently; but that is now being eradicated, regardless of past sacrifices and lives given to defend it; that history is being re-written to Britain's discredit (giving as an example a children's text in which the French are credited with inventing the steam-engine) The DSGB - and others of similar views - need a much broader audience, he says.

A READER draws attention to the new application form for a driving licence, which asks whether drivers can read "a car number plate with figures on it which are 79·4 millimetres high", from a distance of (a) "20·5 metres" or (b) "12·3 metres".

So wonderfully, manically meticulous are these figures that one might conclude, along the lines of Chesterton's essay, *The Mad Official*, that the DVLA had gone quite off its head. The explanation, of course, must be the same as that for the guide book to Berkshire I read some years ago, which described the hole in the famous "Blowing Stone" of Uffington as being "around 46 centimetres long". Boggling at this admixture of vagueness and precision, one realised that some poor soul must have been given the task of "metricating" the guide, which originally had "around 18 inches".

I see, incidentally, that the Laws of Cricket have now been updated, to allow for a cricket pitch to be "20·12 metres" long, and the stumps to be "71·1cm above the ground". I am not sure whether it would please M Delors to see us coming so meekly into line — or whether it would merely confirm him in the belief that we are a people so irrational as to be beyond redemption.

Christopher Booker, writing in the SUNDAY TELEGRAPH, March 1992/1120.

SIR — Andrew Gimson's article (July 6) subjected the metricators to some deserved sarcasm, and rightly implied metrication was an assault on convenience and human qualities.

It is an act of cultural vandalism, designed to obliterate the hard-won and practical modes of thought which enabled Britain to achieve the Industrial Revolution by having a measuring system based on common sense ratio, proportion and human scale that leaves "toytown" metrics floundering.

The metric system is not rational, merely decimal. It is defeated by the housebrick, the simple binary mathematics of kitchen-weighing, the geometry of angle — and hence time — measurement, and by the economics of packaging. It substitutes strings of figures and decimal approximations for straightforward ratios and fractions.

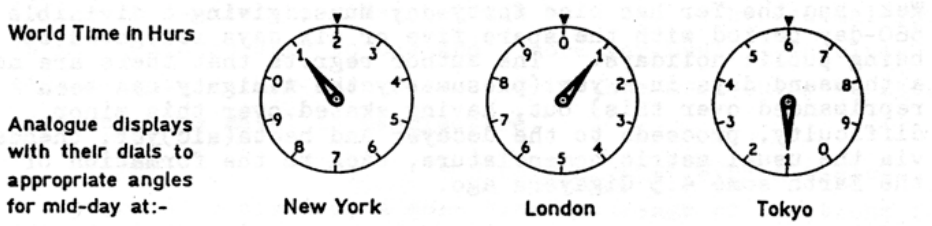
It is astonishing that journalists alone voice opposition to this externally-inflicted disease. Why do we not hear from the politicians or academics? Why are there no scientists prepared to break from their decimal-metric thraldom and consider natural proportion, practical (not laboratory) measurement and even, perhaps, the flawed number-system itself?

DONALD HAMMOND
Denmead, Hants

DAILY TELEGRAPH 1991/1196

ALL THE TIME IN THE WORLD?

Arthur Whillock dissects the latest decimal-time proposal
and philosophizes en route.



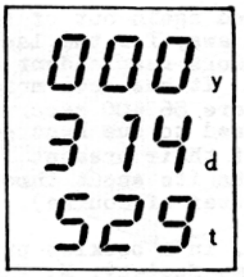
The 0·864-second second has just peeped again out of the wainscoting, where it has been burrowing away for the last two dozen yers (sic). Its latest *imago* is more resplendent than formerly proposed: World Time, no less. It has been noted by many decimal transmogrifiers that there are 86400 seconds in a standard day, but this could be increased to one hundred thousand if they were reduced to 0·864 of their present duration (as the farmer said, once it gets its snout through the fence, the whole pig follows as a matter of course).

We have received details of the above, in a booklet priced £5 from the President of the Decimal Time Society, M. Pinder, BSc., (6 Ramble Close, Warsash, Hampshire, S03 9GT). In his booklet, entitled: 'Time for a Change', Mr. Pinder outlines the many advantages expected to ensue from a time-keeping arrangement whereby time would be the same everywhere on Earth *regardless of the local position of the sun.*

With analogue time display, in which the hour-hand represents movement of the sun, it is reasonable to expect that the hand will be at the top of the dial at midday; but with WorldTime this is possible only in an area containing the datum of the global time-frame. Elsewhere, either the midday 'hur' will have to be clearly marked or the dial tilted to set its zero at this angle. (The contention over which country would have the convenience or honour of an upright dial should be quite diverting.)

The Greenwich Meridian was accepted as a zero for international time zones in a conference at Washington in 1884, since most of the world's shipping then used it. (Conference also voted to continue work towards decimalization of time and angle. - Ed.) France abstained, miffed that Paris was not selected and having had suggestions of Jerusalem or the Great Pyramid turned down (anything to prevent Albion Perfide's acquiring any of La Gloire, undeserved since there had been an observatory at Paris some eight years earlier than at Greenwich - although the English Royal Society was founded four years before the French Academy of Sciences).

To conform with strict S.I., in which multiples of a thousand only are allowed between units, the Day in Worldtime is divided into a million parts, each 0·0864 of a second. Such a brief duration is too short for practical purposes, so a named unit, the Tim, is introduced, a thousand times greater and equal to 1·44 present minutes. The Hur is a hundred tims and the day ten hurs long. There is (of course) a ten-day Wek and the Yer has nine forty-day Muns, giving a divisible 360-day period with the spare five or six days relegated to being public holidays. The author regrets that there are not a thousand days in a yer (presumably the Almighty has been reprimanded over this) but, having skated over this minor difficulty, proceeds to the decayer and hecta(sic)yer; thence, via the usual metric nomenclature, back to the formation of the Earth some 4·5 Gigayers ago.



THREE ROW DIGITAL CLOCK THREE ROW DIGITAL WATCH

For digital display, a three-figure number of time has the first for hurs (2.4 hours), the second for decatims (each 14·4 minutes or about ¼ hour) and the last for unit tims (approx.1½ minutes) with optional changes to millitims for timing fast movements or to days and yers for dates. It is hoped that the time at the end of the millenium will be denoted by:

000y 000d 000t.

Digital displays, however, are going out of favour, their only merit being cheapness. The two-dimensional symmetry of a clock-dial conveys instant information on the passage or availability of time which can be read at a glance and held in the memory for further use without resort to mental arithmetic - not easy when running for a train and dangerous when driving a car.

Worldtime, coldly logical and devoid of character, with no signposts to indicate where you are or going, is certainly a complete abrogation of an important part of our cultural heritage: not even the days of the wek (divided into two five-day periods designated 'first' and 'last.')* have identities other than ordinal numbers - 'oneday', 'twoday' - with the muns treated likewise. There is no mention of seasons (not easy in a nine-mun aggregate). Perhaps it will be our turn next, so that the American jail song: 'Now that my name's a number, a number is my name' sums-up the eventual situation.

*Shouldn't that be 'former' and 'latter'? - Ed.

We may even become superfluous in a computerized paradise dominated by giant machines as described in Olaf Stapledon's 'Last and First Men'...

In his 'Case Against Decimalization' (1960) a founder-member, Professor Aitken, described how superstitious decimalists would sit up on the eve of the coming millenium to await the dawning of a new Heaven and a new Earth. He had no doubt that there would have been incredible technical progress by then, and possibly the transition from a defective system of measurements, but would have been dismayed if told that there would still be attempts to force time - and angle - into the grey, lifeless metric pattern. Time and angle, on their elegantly divisible basis, serve deep-seated needs at all levels of use; so any interference to these for commercial or political expediency (the ruin of science, said Aitken) will create surprising reactions.

To start at Year One is the aim of authoritarian regimes (which only Pol Pot achieved for a brief spell). Reforms of the calendar have often had ulterior motives, chiefly the destruction of previous routines so that social effort could be deflected to new ends. The ten-day week of early civilizations - Sumeria, Egypt, Scandinavia - was re-invented by the French Jacobins deliberately to disrupt religious observances which exercised great control of the populace (the five or six spare days were given the names of virtues instead of local deities as previously).

The Russian five- and later six-day 'weeks' had a more serious purpose, which was to institute a continuous production regimen in factories both for rebuilding and against the next invasion (then considered inevitable). Rest days were circulated round the five days of the week, with the advantage that only one-fifth of the workers were free at a time to discuss the grinding conditions, but it also led to irresponsibility: with a continuously-varying composition of staff, those absent could always receive the blame. A sixth day of rest had to be added for common use, but certainly not the seventh! This lasted till 1940.

All such attempts at social engineering fail by disrupting the need for regularity in our social affairs (Worldtime, as described in the booklet, re-organizes the working population on what can only be called a grand scale). French peasants, for example, stuck to their seven-day marketing routines despite the Republican calendar. A seven-day cycle has existed alongside other divisions of the year as far back as records are available, and has been considered the most suitable for our psychological well-being. We all know that the week is hardly long enough, but would we be able to recall where we were ten days ago? The adoption of a seven-day routine for religious ceremonies might be an acceptance of this rather than vice versa. It may have been owing also to seven's being the best divisor of movements of the sun and moon with least remainder; that there were the eyes of seven Gods watching over us was a decisive coincidence.

Grouping and division by sevens, so frequent in legend and superstition, was particularly strong in Hebrew and Egyptian cultures, which had considerable interaction. There is a plausible theory that early Hebrew chronologies, up to the time of Ahaz, were in base seven... Datings of the Kings of Israel and Judah can be aligned on this assumption, with the zero of the Jewish calendar set at the Flood and the birth of Shem, eponymous ancestor of the Semites.

Worldtime suggests that the same buildings are used for different religions, with days allocated during the wek as shown at right.

Days	0	and	5 - Moslems
Days	1	and	6 - Jews
Days	2	and	7 - Christians
Days	3	and	8 - Hindus
Days	4	and	9 - Buddhists

(Now that's what I call 'Organized Religion'... - Ed.)

There is, of course, a prior claim to the word 'Tim'; It is a small unit of scientific time in the Time, Gravity & Mass (TGM) system of dozenal measures, equivalent to the standard hour divided by a dozen to the fourth power. It renders the acceleration due to Earth gravity as unity by defining a perceptually-valid unit of length close to our present foot. With g = 1, the numerical distinction bewteen mass and weight disappears (to the great relief of students), eliminating a fruitful source of error which occurs when the recommended approximation to the system base (ten) is used in S.I. (By contrast, the FPS approximation - 32 - is a useful binary number whose displacement cannot fail to be noticed) TGM does not involve alteration of the clock-dial or the use of time for everyday purposes.

The Worldtime 'second', the centitim, would require a 16% increase in the speed of electricity generators and the millions of chronometric control motors attached to them via the mains, and we must question whether this can be done easily. Revisions of nomenclature or dimensions are largely paper exercises which can proceed alongside existing standards, but changes to material equipment would be a serious matter (hence, for example, the survivial of the QWERTY keyboard despite attempts to introduce a more ergonomic layout; or the piano keyboard, which needs different fingerings for each key, which endures although an improved arrangement needing only two different fingerings has been suggested). It is difficult to imagine a time when these would be changed.

An eighteenth-century mechanistic view of the Universe is surely the progenitor of the notion that facts can be slotted into an immutable form 'pour touts la peuple, pour toujours'. The Uncertainty Principle, recent discoveries in Chaos and Fractal theories, and now the multi-dimensional 'Super-String' theory of cosmology confirm the suspicion expressed by J.B.S. Haldane: 'Not only is the Universe more complex than we think, but more complex than we *can* think.', which was some admission for him. However, within the feeble thought-processes of mere four-dimensional creatures it has now been determined that the entire Universe is composed of twelve fundamental particles only, arranged into three families of four each.

Life itself has been defined as the mode of existence of protein substance built-up from elements whose valencies comprise the dozenal factor range of 1, 2, 3 and 4 (hydrogen, oxygen, nitrogen and carbon), which permit the formation of an almost infinite range of complex molecules. John Quincy Adams advised the Weights and Measures Committees of the U.S. government: 'Decimal division is not a property of space, time or matter .... Nature has no partiality for the number ten.' Our job is to explain that attempts to force natural phenomena into an inadequate frmework of measures can only result in a makeshift arrangement that cuts across the structure of the real world and inhibits our understanding of it.

Progress, as defined in the Worldtime prospectus, means to preserve the best of what has already been created and discard that which has been superseded; but which are they? Surely, the ones that can properly define the arrangements of the material world, provide the best means to describe it and allow us to contact it with the greatest ease have the better claim? Those methods subservient to a mere accident of biology (efficient as it may be for grasping branches) cannot be important in the long run. Many a 'reform' is no more than a transfer of difficulty from one group to another which is unaware, or is unable to defend itself (decimal currency, for instance, was for the convenience of money-counters to the disadvantage and cost of money-users). This process operates in the sphere of *concepts* as well as in the material region, creating cultural desert areas and calling such results progress. As Tacitus would have put it (with the beneficiaries claiming success): 'The history of any conflict is always written by the victors.'

Standardization is advantageous at some levels - chiefly the very large and the vary small - for an overall frame and for scientific work. Something analogous to a rockery made from a regular structure of blocks with gaps between for the flowers might be desirable. Variability is essential if we are not to sink into an undifferentiated mass of Zombies who have no knowledge of their past, no care for their future and to whom only the latest instruction on what they should be thinking has any reality. One of Karel Capek's characters in R.U.R., when surrounded by Robots gathering for the final assault, declared that it was a mistake to have made them all looking alike; he should have said 'thinking alike' as well. As for travel, when all places are the same, all bazaars display the same tourist tat and (courtesy of Worldtime) it is not even necessary to alter one's watch anywhere, there will be nowhere worth visiting.

The malaise that now affects all urban societies must be partly owing to the wide gap between the majority of their members and the processes which support them. Knowledge of the technology involved is available only in terms not in accord with ordinary experience. People have become alienated from the natural world, which is now no more tangible than pictures on TV, and. fast disappearing (one life-form becomes extinct every day). A digital time-display, particularly if it is dissociated from ambient circumstances, is part of this process. A complete decimalization of everything would be its apotheosis. The question to be asked is, as always, 'Cui-bono?'.

It would be unfair to impute devious motives to decimal-metricists. No doubt most of them believe that theirs is the only means of obtaining international agreement on calculation and measurement; their salient fault is the fundamental one of 'better sure than right'. Technical rationalization, however, intended for purely practical purposes, inevitably acquires political dimensions because of its potential for enhancing economic and social control. This is true with any all-embracing system; but one which comprises arbitrary concepts within an abstract context is unlikely to allow effective resistance to unwelcome moves. Ideas can be diverted, either deliberately or unconsciously to ends quite at variance with the ideals they were intended to foster. The possibility of restricting freedom of expression in any form is never overlooked by interested parties.

Saddled as we are with an inadequate numeration and calculation system, no amount of fiddling with ten-based arrangements can create the conditions needed for a further advance. Attempts to do so, concurrent with legal suppressions of extant methods, confine the future to prevailing narrow dogma. It should be a cardinal responsibility not to destroy an inheritance that we do not yet know how to use properly. The Dozenal Society must continue to voice the resentment and frustration felt by many at the steady elimination of common practices by unauthorized Orders in Council; and to reassure people that their customary measures are not as unscientific as they are being led to believe.

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A.F.W

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Footnote (*1200/2016) - there's no sign of the WorldTime Society on the Web, but there are other decimal time enthusiasts out there ...

EDUCATIONAL COMMENTS

From 'GEOMETRY JUNGLE', a schools' mathematics text

The word 'protractor' is dropped, the device now being called an 'Angle-Measurer'.

A die is now called 'a dice'.

Regular polygons are drawn by pupils only with the said 'Angle Measurer', never by construction.

The regular dodecagon is just briefly mentioned (not by name, but as 'you could use twelve sides'), whereas hexagon, octagon and decagon are illustrated and emphasized.

Introduction to the topic

'Over the next seven weeks you will be travelling through the Geometry Jungle. Shortly you will arrive at the 'Landing Point' in the bottom left-hand corner of the map. You are on a quest to retrieve the golden sphere. This is a solid ball of gold which was stolen from your people by the evil polygons. The polygons are a set of many-sided shapes who will cause you some problems throughout your journey.'

No, this is not material aimed at seven- or eight-year olds, but is provided for secondary-school pupils aged thirteen...

The 1991 SMP Maths. Paper suggests that there may be forthcoming a £5 coin in the shape of a pentagonal curve of constant width (CCW).

There are no 'number-base' questions in the entire exam. No questions involving weeks/days or minutes/seconds appear. One very small clock-time question uses a gap between hours and minutes (13 15). Front-cover instructions for the examination give times using a decimal point between hours and minutes.

BRANSON'S BALLOON

The Educational TV programme, 'Mathsphere' (BBC2), focused on Richard Branson's Pacific flight in a hot-air balloon. The German-sounding engineer/navigator gave the capacity of the balloon in cubic feet, then obediently translated it into cubic metres... During the flight, Branson and the navigator talked in terms of feet/min. for climb- and descent-rates, knots for speed and nautical miles for distance, of course. The programme presenter, however, insistently rendered these distances in kilometres, even stating at one point that one degree of Latitude is equal to one-hundred-and-eleven kilometres (never mentioning that it is actually sixty nautical miles).

PSI Albion 1. SI Metrique 0

The ITV programme "Scientific Eye" demonstrated the idea of pressure by resting the weight of some bags of sugar on a one-inch wooden cube, which in turn rested on the tyre of a bicycle wheel. The tyre began to squash when the appropriate number of bags was used. Thus, the notion of "pounds per square inch" was clearly shown.

One can only guess at the pain caused to the producers of this otherwise rigidly metric series: having to use a square-inch unit must have been bad enough; also having to give the weight of bags of sugar, which used to be 2lb. but have now been metricated to 1kg., as "about 2lb." would have been pure anguish. A further point to be relished was the use of a *gravitational unit* - the Lo. which is readily understood by ordinary people but usually forbidden by scientists.

So, why did they do it? Well, the SI pressure unit is the Pascal, which is 1 Newton per square metre. A Newton is that force which will accelerate a mass of 1 kilogramme at a rate of 1 metre per second per second. Tyre pressures are supposed to be given in *bars*: one bar equals 100 kilopascals.   Got it? Now try to explain that to a child of no more than average intellect and devise a simple demonstration, using familiar objects and materials, of what is meant by a tyre-pressure of, say, 1·8 bar...

No wonder that the Industrial Revolution was brought about by *practical* men in Britain and the USA, unhampered by abstract absolutism and so building their steam-engines to work on - pounds per square inch!

****

HISTORIC MEASURES OF ENGLAND

WINCHESTER, once capital of Wessex and seat of government of the Saxon kingdom of Anglia, was the repository for national standard weights and measures and was associated with London - a rising centre of commerce - up to the Tudor period. No samples of Saxon measures remain. Their values are deduced from documents and grave-goods.

William the Conqueror wished to be regarded as Edward the Confessor's lawful successor and so stated that measures and weights ".. most trustworthy and duly certified ..." should be "... exactly as the good predecessors have appointed.".

Later commercial and political revisions decreed destruction of the old standards out this, happily, went unheeded by some on the administrative periphery. Winchester Museum holds many originals, including Edward III's haber-de-pois weights (still valid today), Henry VII's and Elizabeth's capacity measures and their standard Yard - all invaluable as research anchor-points.

The Museum has issued a booklet illustrating the measures held and summarizing their history. It is available for £1 post paid from The Historic Resources Centre, 75 Hyde Street, Winchester, Hampshire S023 7DW.

A.F.W

SWITCHCRAFT

The extract below is from a text-book on practical electronics.

(iv) Rotary wafer. One or more discs (wafers) of paxolin (an insulator) are mounted on a twelve-position spindle as shown in Fig. 9.31(a). The wafers have metal contact strips on one or both sides and rotate between a similar number of fixed wafers with springy contact strips.

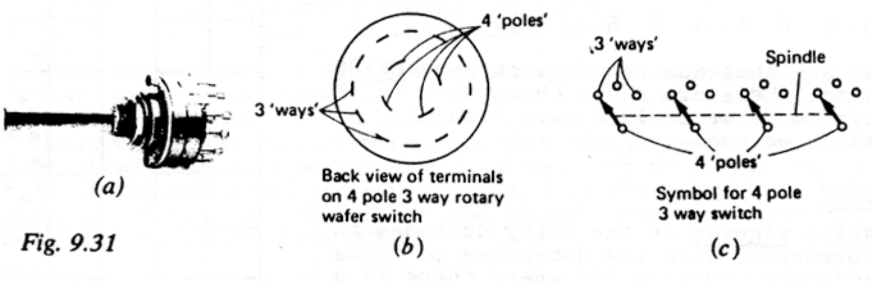


Fig. 9.31

The contacts on the wafers can be arranged to give 1 pole 12 way, 2 pole 6 way, 3 pole 4 way, 4 pole 3 way (as in Figs. 9.31(b) and (c)) or 6 pole 2 way switching.

These switches are mass-produced and in universal use as selectors on the panels of oscilloscopes, etc., even where the decimal-minded want a scale of ten to select from (they ignore two terminals). It is as good an illustration as any of the versatility of the dozen. Decimal enthusiasts may be challenged to design a switch with comparable adaptability using ten outer terminals instead of twelve.

****

ROADCRAFT

As the last filling-stations go metric by order of Big Brother in Brussels, here are some lines for the remaining few who keep track of fuel-consumption ...

Imperial petrol? Easily done:
    TWO gallons are just nine-point-one;
If FOUR are what your tank is due,
    Then cut off at eighteen-point-two.
It now is clear and plain to see:
    SIX are twenty-seven-point-three;
If EIGHT are needed, take some more
    But stop at thirty-six-point-four.
TEN gallons - for a distant drive
    Are in at forty-five-point-five;
For TWELVE these anti-metric tricks
    Come right at fifty-four-point-six...

Or, of course, memorize the first two lines and multiply up.

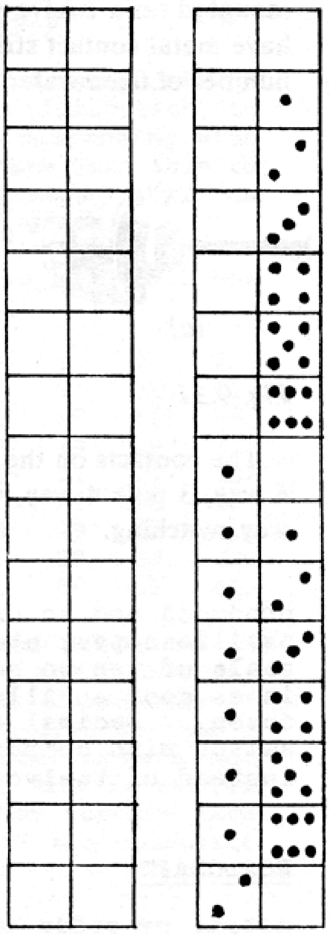
NUMBER-BASES (ii): WRITING NUMBERS IN DIFFERENT BASES

Another article for beginners - Part (i) appeared in JOURNAL 3 - which can be used by members who wish to teach someone (perhaps a child: children are no longer taught this at school) the rudiments of the topic.

Dominoes, at least the ordinary kind, embody the numbers one to six encoded in dot patterns and a blank (zero).

0 1 2 3 4 5 6

We say that dominoes are in base <u>SEVEN</u>, since there are seven characters in the system if we include zero (nought), which we must.



<u>WORK</u>

Write <u>figures</u> in the empty dominoes to correspond with the dot-codes on those adjacent, putting 'O' where there is a blank in the right-hand square.

• • • • • • • • • • • • •

What you have now done is to write the first fourteen numbers in base seven. In base seven, 'fourteen' is shown as two lots of seven and no units over. We should describe this number in base seven as 'two-nought'.

Hence, in base seven, 2 x 10 = 20

and 5 + 6 = 14

and 3 x 4 = 15

and 13 ÷ 2 = 5 etc.

We note further that seven in base seven is written "10" and realize that <u>any number written in its own base will be "10"</u>.

Try now to write numbers to the base seven in successive rows in the grid below, remembering that each time you get to a multiple of <u>seven</u>, you must "carry one" to the left. Some of the numbers <u>have</u> been put in to make it easier to avoid mistakes.

Start here →

1			4			10
					16	
		23				
				35		

Can you think of anything else you could write in this grid?

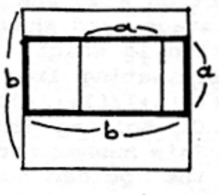
(TO BE CONTINUED)

MATHEMATICAL SECTION

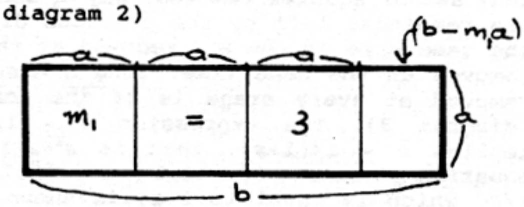
Continued Fractions - A Visual Approach by George Jelliss

Squaring the Rectangle. The ratio $s = a/b$ of the shorter to the longer side of a non-square rectangle can be regarded as expressing its “shape” or “squareness” or “proportions”. (We could also use the ratio $b/a = 1/s$, but I prefer to deal with fractions between 0 and 1.) The number s also expresses the ratio of the area of the rectangle to the area of a smallest square containing it, also the ratio of the area of a largest square contained in it to the area of the rectangle. i.e. $s = a/b = a^2/a\times b = a\times b/b^2$ (diagram 1).

(diagram 1)



(diagram 2)



Any non-square rectangle can be divided up into squares by the process of removing from one end of it a square of the largest possible size (a), then applying the same procedure to the smaller rectangle that results. If the numbers of squares removed in this way, of the first, second, third, etc, sizes are m_1, m_2, m_3.. what is the ratio of sides of the rectangle, expressed in terms of these numbers? The answer is $a/b = 1/(m_1+1/(m_2+ 1/(m_3+...)))$ [or $b/a = m_1+1/(m_2+1/(m_3+...))$]. This is easily proved as follows: If from one end of a rectangle of shape $s = a/b$ we remove m_1 squares of size $a\times a$ to leave a rectangle smaller than $a\times a$, than the shape s of the smaller rectangle is $s_1 = (b-m_1\times a)/a= (b/a)-m_1$ (diagram 2). Thus we find $s = 1/(m_1+s_1)$.. We can now similarly express s_1 in terms of m_1 and s_2, giving $s = 1/(m_1+ 1/(m_2+s_2))$, and so on. The expressions resulting are known as “continued fractions”.

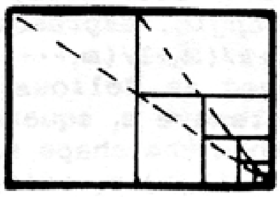
Rational and Irrational Numbers. In practice the dissection process always terminates after a number of steps because the squares get smaller and smaller until they go beyond the accuracy of our measuring or drawing devices (or beyond the physical limits of observability, as expressed in Heisenberg’s uncertainty principle). In the fantasy world of theoretical mathematicians however the process may be supposed to go on indefinitely in some cases. If the process terminates then the ratio is termed “rational” while if it goes on for ever it is “irrational”. This is one of the neatest ways of defining the distinction between rational and irrational numbers in theoretical mathematics. Rational numbers suffice for all practical purposes.

Ratios. A ratio (non-zero positive rational number) s can always be expressed uniquely in the form $s = p/q$ where p and q are numbers (non-zero positive integral numbers) with no common factor, i.e. it is expressed as a “ratio in its lowest terms”. It is important to realise that given any such ratio s the numbers m_1, m_2, m_3,... in its continued fraction expression are all uniquely determined, and conversely that if any numbers m_1, m_2, m_3 .. are chosen (the last not being 1) then they determine a unique ratio $s = p/q$, i.e. the continued fraction is a “representation” of the ratio.

Convergents. If we calculate the series of continued fractions

$1/(m_1 + 1/(m_2 + \ldots\ 1/m_r)) = p_r/q_r$, for $r = 1, 2, 3, \ldots, n\text{-}1$ we get a sequence of ratios that are called "convergents" to p/q. These are alternately larger and smaller than p/q and get closer and closer to p/q, in fact $|\,p_r/q_r - p/q\,| < 1/2q_r^2$. Moreover the convergent p_r/q_r is the best approximation possible to p/q with denominator q, or less. In fact if $|\,p'/q' - s\,| < 1/2q'^2$ then p'/q' must be one of the convergents to s. Any convergent can be calculated from the preceding two convergents and the new term m_r by the recursion relations: $p_r = m_r \times p_{r-1} + p_{r-2}$ and $q_r = m_r \times q_{r-1} + q_{r-2}$.

The Divine Proportion. It is natural to ask: is it possible to have $m_1, m_2, m_3, \ldots$ all equal to 1? As noted above, in an exact dissection the last set of squares removed, say m_n, cannot be 1, since this would mean the remainder left by the previous dissection step was a square of the same size as the m_{n-1} removed at that stage, and should have been removed at the same time. Thus a dissection in which one square is removed at every stage is of the non-terminating irrational type, (diagram 3). The expression $s = 1/1+1/(1+1/(1+ \ldots)))$ evidently implies s - $1/(1+s)$, that is $s^2 + s - 1 = 0$. Solving this quadratic equation gives $s = (\sqrt{5}-1)/2$ = *0'74↋↋67... This number (or its inverse $1/s$, which is equal to $1+s$) is known as the "golden number" or the "divine proportion" or the "mean and extreme ratio" among other soubriquets. The convergents to this ratio are the ratios of successive numbers in the Fibonacci sequence: (dozenal notation) 1/1, 1/2, 2/3, 3/5, 5/8, 8/11, 11/19, 19/2↊, ... (see DJ8, p.*17]

(diagram 3)  (diagram 4) 

The DIN Proportion. Some of the traditional paper sizes were based on an approximation to the divine proportion. For example "royal" *21 × *18 inches (shape ratio *0'97) when cut in half successively gives "royal demi" *18' × *10'6 (shape ratio *0'76), "royal quarto" *10'6 × ↊ (*0'97) and "royal octavo" ↊ × *6'3 (*0'76). Cutting a sheet in half alters the shape s to $\frac{1}{2}s$. The current standard international paper sizes, deriving from the German DIN ("Das ist Norm") standard, are shaped so that $s = \frac{1}{2}s$, that is $s = 1/\sqrt{2}$ (diagram 4).

Lagrange's Theorem. Lagrange proved the general theorem that the numbers in a continued fraction recur periodically if and only if the ratio expressed is a quadratic surd, i.e. satisfies an equation of the form $A \times s^2 + B \times s + C = 0$, where A,B,C are whole numbers (+, - , or 0).

For example, if we remove two squares at each step we have $s = 1/(2+s)$ and so $s^2 + 2s - 1 = 0$ which gives us $s = \sqrt{2}-1$. The continued fraction for $\sqrt{2}$ is thus: 1+1/(2+1/(2+1/(2+...

If we remove alternately 1 and 2 squares, $s = 1/(1+1/(2+s))$ whence $s^2+2s-2=0$ and $s = \sqrt{3}-1$. Thus $\sqrt{3}$ = 1+1/(1+1/(2+1/(1+1/(2+... If we take 2 and 1 squares alternately we get similarly $s = \sqrt{3}+1$.

From the expression for the divine proportion we deduce that $\sqrt{5}$ = 1+2/(1+1/(1+1/(1+ ... but in standard form (in which the numerators are all 1s) we find: $\sqrt{5}$ = 2+1/(4+1/(4+1/(4+...

Using a Calculator. If we have a decimal value, say the exponential number e = 2·7182818, that we wish to express as a continued fraction, we repeat the following two operations: (a) write down and subtract the integral part (b) take the inverse of the fractional part. Thus we get 1·3922113, 2·5496464, 1·8193515, 1·2204774, 4·5356121,... and the integral parts show that e = 2+1/(1+1/1(2+1/(1+1/(1+1/(4+... In this case the (dozenal) convergents are: 2, 3, 8/3, Ɛ/4, 17/7, 73/28.

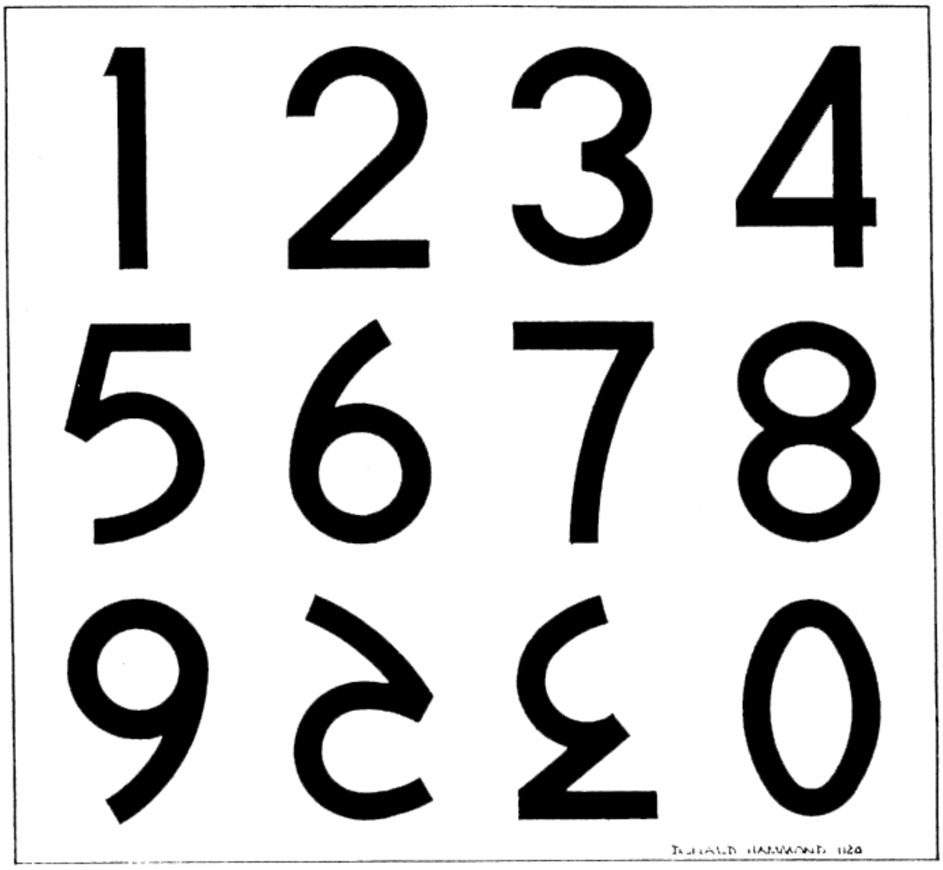
The Hammond Temperature scale. In Dozenal Journal #1, p33' Donald Hammond described a "Rational Fahrenheit" absolute temperature scale (better called the Hammond scale, I suggest) with the ice point at *350 and the steam point at *480. He did not mention that the ratio *350/*480 = *35/*8 in lowest terms, is one of the convergents to the continued fraction for 273·16/373·16 = 0·732018 = *0'894Ɛ (the ratio of the two temperatures, which is the same expressed in any absolute scale). This indicates that no better fit is possible.

Approximating Pi. Working backwards from a figure for π correct to sufficient places of (denary) decimals π = 3·1415926536 can be expressed as the continued fraction: = 3+1/(7+1/(13'+1/(1+1/204'+ .... which gives the sequence of (dozenal) convergents: 3, 1Ɛ/7, 239/8Ɛ, 257/95, 50221/171Ɛ6. The figure 335/113 (*257/*95) is said to have been obtained by a Chinese astronomer Tau Ch'ung-chih as early as c.450 AD, and 22/7 (*1Ɛ/7) was of course known to Archimedes c.250 BC.

Degrees and Radians. The above convergents to π enable us to work out the best unit in which the degree and radian can both be expressed in whole numbers. The answer is rad/7Ɛ5Ɛ' = 1/180'. This unit is simply the angle of rotation of the Earth in one *second of time* and is equal to 13' *seconds of arc*. This is accurate to five dozenal figures: rad = *7Ɛ5Ɛ/*180 = *49'3672" (correct is *49'3671). Accurate to three figures is the relation: rad/*967= 1/*20, which gives rad = *49'36.

Years and Days. Another pair of measures difficult to reconcile are the day and the year: (mean tropical) year - 365·2422 (*265'2ƐƐ6) day. As a continued fraction we find: *265+1/(4+1/(7+1/1+ .... with the (dozenal) convergents: 265, Ɛ19/4, 6168/25, 6Ɛ85/29, 2307Ɛ/Ɛ8. The last is specially interesting as *Ɛ8(128) = 2^7. This shows that in an accurate calendar *Ɛ8 years = *2307Ɛ days, so we must insert an extra *27 days every *Ɛ8 years. (i.e. 1 leap day every 4 years except on the *Ɛ8th year). This also suggests that we should count historical time in units of 2 years rather than centuries. Looking at the result the other way round we find that the largest unit in which both year and day can be measured with an exact number of units (to four decimal place accuracy) is year/*2307Ɛ - day/*Ɛ8 = *Ɛ'3 (11·25) minutes, which does not fit in very well with our customary division of the day.

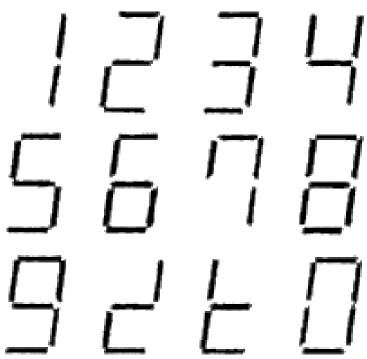
Distances in the Solar System. Another application of the method is to compare the Earth-Sun and Earth-Moon distances. These are *2716Ɛ000 miles and *Ɛ6288 miles (mean centre-to-centre values), and their ratio is *285'20889 = *285+1/(5+1/(1+1/(4+1/(1+1/(1+1/(1+1/(*47+..., with (dozenal) convergents: 1162/5, 1427/6, 6646/25, 767Ɛ/2Ɛ, 124Ɛ7/54, etc. The denominator 54 = 2^4 is attractive and suggests trying the unit: Earth-Sun/*124Ɛ7 = Earth-Moon/*54 = *21Ɛ0'16 (3732·125) miles = *3586'33 (6006·273) kilometres. This unit is slightly less than Earth's radius: *2363'41 (3963·34) miles equatorial, *2351'ƐƐ (3949·99) miles polar.



Above is a constructed version of the set of digits used in the JOURNAL. They are drawn to a ratio of 5 : 8 width-to-height, which approximates to the Golden Section.

Below is the 7-segment set.

LED 7-segment display ‘tiles’ are commercially available: would any electronics buff care to make a dozenal adder?



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WILL BE WELCOMED.

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This is a non-profit-making Society, with membership open to all, whose ultimate aim is the conversion of the base of numeration from the present decimal (ten-count) to a dozenal (twelve-count) radix.

It is also a prime concern of the Society that the mathematical, scientific and technical education of our children should be as broad as possible; should be free of political indoctrination and censorship; and should impart a sound understanding of number, with modes of arithmetic and measurement which are efficiently harmonious with human needs and the unalterable laws of the Universe.

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This has broadly the same objectives as above and maintains a cordial and constructive relationship with the DSGB

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