THE

δozεηιτε

SYSTEM OF NUMBER RECKONING

contents

The

δozεηαl

System

the concept of radix

It can be said that mathematics as a discipline had come of age merely a few centuries ago, at which time the Hindu-Arabic numerals came to be used as a standard numerical notation around the world. That shift was indeed a conceptual leap from the then prevalent and rather unwieldy roman numerals, primarily because it introduced a perpetual system of representing any number using a limited number of symbols. This system is lauded among the mathematical fraternity as a 'Place-Value' system.

The concept of place value is ingenious in that it optimizes the use of certain select symbols and takes advantage of the unbounded nature of space, thus being able to note any number as humongous or minuscule as one wants. Every number is composed of one or more symbols arranged in a line. The location of each symbol relative to the others is known as its 'place-value', and the symbol that inhabits any particular 'place' is called a 'face-value'. The place-values represent successive exponential powers of a certain number which is termed as a 'base' or, more technically, a 'radix'. The face-value that inhabits a particular place-value indicates the number of instances of that place-value to be taken into count. It can be seen that the number of face values, that any place in the number can use, is limited to the numerical value of the radix. Thus the infinitely many possible places need face-values from only a finite selection of symbols, in order to represent a number of any magnitude. These face-value symbols are thus the only requirement (apart from the fractional point) for the notation of any number.

For any numeral system based on a place-value system to work, the value of the radix has to be predetermined as a standard, so that everyone understands the numbers they read. The modern world has standardized this radix to be 'ten' which is why the numbers we use today are said to be of a 'decimal' system. The choice of ten as a radix is presumably an offshoot of the human habit of counting with fingers, which by biological design, are ten in number (including thumbs). This also lead to the term 'digit' (which refers to fingers or toes) to mean the face-value in any place within the number since there are ten such symbols to accommodate the radix of ten.

The digits as we know them, run from 0 to 9, but they don't always have to. The value chosen for a radix is just a matter of convention for the use of numbers in any sphere of study. The decimal system, while being the most widespread, is not the only radix used in the modern times. The world of computers is governed by numbers of radix two, commonly known as 'binary', since it lends itself easily to logical manipulation, and as the name suggests, there are only two digits used: 0 and 1. There are also derivatives of binary based on powers of two which programmers would be familiar with by the names of octal and hexadecimal systems.

ραδιχ τωεlvε (δozεηαl)

Perhaps the most commonly recognized radices are the Decimal and the Binary systems, but they are not necessarily the most optimal for arithmetic. In the history of technological development in mankind various radixes have been used by different civilizations. For example, the Mayans are known to have used base twenty and the Sumerians, base sixty. Vestiges of the Sumerian system can be seen in several modern conventions like the sixty seconds in a minute and sixty minutes in an hour (or in a degree if you are measuring angles). Three hundred sixty degrees form an entire circular revolution. However there is one radix that seems to optimize between the cumbersome though comprehensive notations of these bases and the lightweight yet in-discrete nature of bases like binary and decimal systems, and that is the duodecimal alias 'dozenal' system or radix twelve.

Several mathematicians have seen a promise in the use of radix twelve, the dozenal system, because it is a highly divisible number especially for the most commonly used multipliers and ratios in day-to-day life (i.e. two, three, four, half, a third and a quarter). At the same time it is not cumbrous like other highly divisible but huge base numbers mentioned previously, remembering whose digits would give a headache. In addition, the number twelve seems to be a recurring theme in natural phenomena like the twelve months of a year or the twelve semitones of a musical octave, not to mention its convenience in commercial spheres where slices in a loaf of bread or eggs in a carton are normally grouped in twelves. There are, in fact, Dozenal Societies who aim to spread awareness and design systems and conventions in favor of the dozenal system, the most popular of which are the Dozenal Society of America (DSA) and the Dozenal Society of Great Britain (DSGB).

As with any venture, there are a few problems that the Dozenal Societies face when they plan to implement radix twelve as a universal standard for a base system. The astute reader would have noticed by now that since any system with a certain radix requires as many digits as the radix number to accommodate it, the dozenal system requires twelve digits. The choice of these digits has been a hot topic of discussion for several decades in the dozenal societies. Most members seem to prefer retaining the existing ten digits of the decimal system for zero to nine and add two fresh symbols for ten and eleven. Both DSA and DSGB have taken this step, but they have used a different choice of two digits for ten and eleven. No one has yet come to a consensus on the choice of digit symbols for dozenal use around the world. Even the nomenclature of these new digits comes as an issue with the package.

Notwithstanding this difficulty in implementing dozenal use worldwide, apart from public surmounting the mental barrier of using a new system, there is no doubt from the mathematical standpoint, that this system is sought after for ease and utility.

the digit cycle

The Dozenal system lends itself to the use of a concept called the Digit Cycle. This is not unfamiliar to most people since, in the context of twelves, the digit cycle is ubiquitous. It is just a way of arranging the digits on the circumference of a circular dial so as to make it easy to count periodic phenomena in a particular base system, which in this case is dozenal.

Perhaps the most common instance of the digit cycle is the analogue clock with twelve hours to represent half a day as an entire revolution (of the hour hand). This could also be used for denoting twelve months of a year on a circular calendar or twelve five-second blocks of a minute on a stopwatch dial. However not all uses of the digit cycle pertain to time measure.

The twelve semitones of a musical octave can be placed on a digit cycle, or even the three primary, three secondary and six tertiary colors as a standard color wheel. The digit cycle can be used for many purposes in this fashion to organize cyclic phenomena onto a simple dial-like format, and the **dozenal** digit cycle has been found to be particularly convenient.

It should be noted that digits of the dozenal system can be categorized based on the digit cycle. The digits for zero and six lie at opposite poles at the top and bottom of the digit cycle respectively. These are called the pole digits/numbers. The numbers immediately adjacent to the poles are one, five, seven and eleven. They are referred to as meridian numbers. Note that these meridian numbers are all prime (although the prime nature of the number 'one' is debatable), and all primes in the dozenal system (except two and three) end in one of these meridian numbers. Thus they are also called 'meridian primes'. In contrast 'two' and 'three' are referred to as 'base primes' since they are the prime factors of the base number twelve. There are two numbers called triad medians, four and eight, since they form a 'triad' with zero as will be discussed shortly, And two numbers called invert base numbers, nine and ten since they can be obtained by subtracting the base primes from a dozen.

It is worth noting the concept of 'triads' and 'quads'. As seen earlier, zero, four and eight form a triad because the three numbers are equally apart on the digit cycle. Note that they have a common difference of four digits. There are three other triads on the dozenal digit cycle: [one, five, nine] & [two, six, ten] & [three, seven eleven]. Similarly the quads are groups of four digits equally spaced out on the digit cycle. There are three quads: [zero, three, six, nine] & [one, four, seven, ten] & [two, five, eight, eleven]. As per convention a median triad and a median quad are recognized, both including the digit zero. To enumerate, they are [zero, four, eight] & [zero, three, six, nine] respectively. Every digit can be pinpointed with a specific combination of a triad and a quad as will be seen later on in the document.

In the appendix section, four standard dozenite digit cycles have been provided keeping in mind that it is easy to follow and fit all ideas together after all the explanation is done.

The

δοzεηιτε

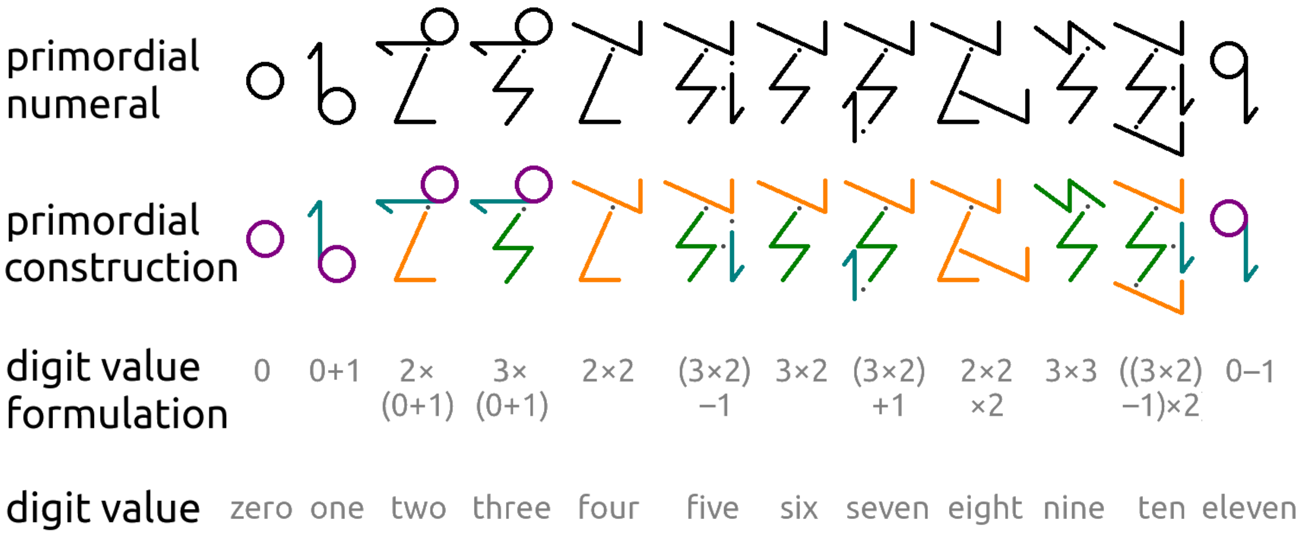
Reckoning

primordial numerals

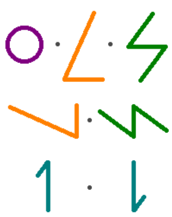
The Dozenite system, as proposed by me is primarily a system of dozenal digit symbology and nomenclature, but it also gives way to a perspective on dozenal arithmetic peculiar to this system. The Dozenite numerals have been developed from a simple set of logical rules and then refined to form digits which, by some serendipity, have a close resemblance to the existing Hindu-Arabic numerals, but as though with two new symbols added **amidst** those of the digit sequence and not the end (as the Dozenal Societies are wont to doing).

The rules underlying the construction of Dozenite numerals are elementary. The base primes two and three are crucial to the formulation of several digits since they are direct multiples of these two primes. There is an elementary symbol each for two and three, which have two and three joint lines respectively. Turning these symbols anticlockwise by a right angle makes them multipliers of their respective cardinal values. There is a symbol for zero/dozen that looks simply like a loop. In addition to these, there are incremental symbols with the function of adding or subtracting one from a digit's cardinal value. Together these elementary symbols combine in different ways to form the twelve digits of the Dozenite system.

PRIMORDIAL NUMERAL CONSTRUCTION



the functions of the shapes in order of precedence:



zero ; two ; three	0 ; 2 ; 3
into two ; into three	×2 ; ×3
plus one ; minus one	+1 ; −1

notes:

> '0-1' is taken to be eleven since it is equivalent to adding a dozen (the base number) to '-1' (0-1)

> in the digit for ten, the second '×2' is placed under the rest of the numeral, thus it takes last priority in its precedence

In this image a ‘digit value formulation’ is given along with every digit that has been constructed. This is just an algebraic translation of the arrangement of the elementary symbols to develop the numeral and it is equal to the cardinal value of the numeral digit. This simple logic designed to construct numerals shapes tailored for radix twelve is comprehensive, for just by looking at the individual strokes in the shape of each numeral, its cardinal value can be ascertained. It might be helpful to provide an enumeration of how each digit has developed from the elementary symbols.

Perhaps the most apparent numeral is zero since it is formed solely by the loop symbol of zero. The elementary base prime symbols two and three aid in forming several digits on their own like four (two into two), six(three into two), eight(two multiplied thrice) and nine(three into three), but the digits for two and three themselves have each a multiple of ‘one’. The two pole digits zero and six are significant since they represent two opposite phenomena; zero is the absence of both base primes and six is the presence of both. These poles are also crucial in developing four other digits, the meridians, by either adding or subtracting one from the pole digits using the incremental elementary symbols: Seven is 'six + one', Five is 'six – one', One is 'zero + one' and eleven is 'zero – one'. The last of these is considered as such because going one step back from zero on the digit cycle reaches you to eleven. The most complex digit in the Dozenite system is that of 'ten', which is formed by multiplying five by two. In this case the multiplication by two is a symbol placed below the existing symbol of five to imply that this is the last operation in formulating the digit i.e. it is performed **after** developing the digit of five.

The digits of the dozenite system are, however, not the only symbols that it uses. The fractional point (equivalent of decimal point) is not a full-stop placed after the unit digit of the number but a dot-like accent placed **above** the units digit. Furthermore, to represent a repetitive sequence of digits in the fractional expansion of a number (equivalent to decimal expansion), a diaeresis mark is placed above the first digit of the repetitive sequence. These symbols are an augmentation to the existing notation of numbers.

This scientific approach to designing dozenite numerals is also intuitive for the reader as can be seen above. However, as you might have observed there is a drawback to using these numerals due to the mere fact that it lacks a certain neatness and refinement. You might wonder how such a set of grotesque symbols could be used for practical notations and arithmetic. The fact is that dozenite numerals can be used without causing any aesthetic compromise, but not in the form that they are in now. The numerals which have been constructed now from elementary symbols are referred to as 'primordial' dozenite numerals since they require more work to become practically functional.

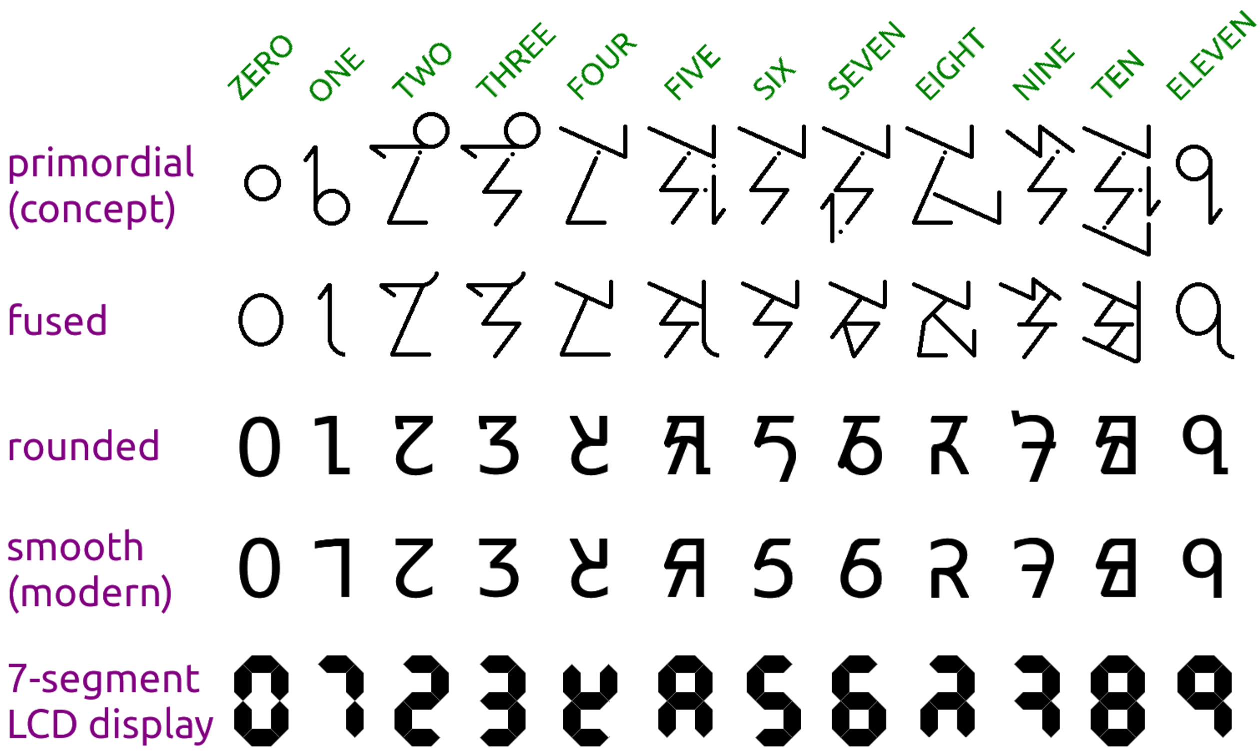
As will be shown in the next section, these numerals have undergone a process of refinement in stages to reach a modern and elegant form which can be used for practical purposes. The only thing to be aware of is the fact that many of the refined forms look similar to Hindu-Arabic numerals that they are not, so make sure you read your dozenite numbers cautiously.

refining the numerals

Although the numerals developed from a simple logical procedure as seen above convey the numerical values of their respective digits faithfully, they are not well-suited for practical use due to their unavoidably grotesque nature. Glyphs and characters of languages around the world get refined with time to come to a modern and usable form. The dozenite symbols have undergone a process that is no different except for the fact that the whole process spanned only about one year of time (and not aeons).

The primordial digits had one major problem that their parts were separated by small gaps, so it was easy to confuse digits that were placed close by. The first stage of refinement was to fuse these fragments so that each digit is a single shape. However, the numerals were still hard to write quickly, for they had too many straight lines and almost no curves. For this reason the digits were rounded and then eventually smoothened out to get the modern dozenite numerals. After this whole process, the digits became aesthetic and usable. The image given below summarises this process of simplification and includes the later developed 7-segment versions of these numerals for LCD type displays.

Refinement of Dozenite digits:



digit nomenclature

Having designed a new symbology based on a radix which is presently not in widespread use, there is presented an opportunity to develop alongside it a system of nomenclature that is simple and distinct from the existing one. The dozenite nomenclature is proposed along with its numerals as a befitting system of vocalising dozenal numbers in dozenite.

In comparison to the existing method of naming numbers, the dozenite system is far simpler to remember and use. It is also more universal due to its use of rudimentary phonetics that typically any culture around the globe can pronounce and understand with ease. There are no special terms for powers of twelve unlike the existing international system wherein every third power of ten has a special name to be memorised which can be itemised as: thousand, million, billion, trillion, quadrillion, quintillion etc.

Nomenclature of digits and numbers:

DIGIT 

UNIT FORM	zen [zen]	doe [doe]	ren [ren]	thir [thur]	four [fore]	quin [kwin]	jeth [jeth]	seph [sef]	echt [ekht]	noe [no]	beth [beth]	leph [lef]
AFFIX FORM	*zy- [zuy]	do [duh]	re [reh]	ith [ith]	fo [foe]	qui [kwi]	je [jeh]	se [seh]	ech [ekh]	no [nuh]	be [beh]	le [leh]

notes:

> unit form is used only for digits in units place or standalone single digits
> affix form is used for digits in any place other than units (including fractionals)
> when there is a fractional set of digits, the terminal digit is in affix form
  and is suffixed by '-m'. the fractional point is placed above the unit digit
> if there is a recurrent sequence of fractional digits, the first digit of that
  sequence is prefixed by 'myr-' and a diaeresis mark is placed above its digit
> the affix form of zero (i.e. zy-) has the following function. when it prefixes any
  number (natural), it means as many zeroes as the natural number are to be placed
> note: zy- + doe = zed

e.g: jezyfour lechnoquin ithre myrdobesem = 

Every digit has only two forms with respect to its name, the unit form and the affix form. The unit form is used only for units digits or standalone digits like those in a telephone number. The affix form is used for digits in any other place whether exponential or fractional. Every name for a digit, whether in unit or affix form, is a single syllable. Also, if the prefix 'a-' is placed before a number it becomes an ordinal value.

When there is a fractional point in the number it does not go by any sound in pronunciation; the presence of a digit in unit form implies a fractional point for any affix digits placed after it. The convention in writing is to place the fractional point not as a full-stop, but a dot above the unit digit. Furthermore, if a repetitive sequence of fractional digits occurs in the fractional expansion, it is prefixed by 'myr-' and denoted by placing a diaeresis mark on the first digit of the repetitive sequence. These details have been summarized in the given image.

Also note the function of the affix form of the zero value 'zy-'. Unlike other affix forms this is a prefix which cannot stand alone. Prefix it to any positive integral number and it means as many zeros as that number are to be placed in that region of the number. This area in the number can be anywhere before or after the unit digit but not through it. If in a sequence of zeros ends in the units place it is treated as a single stretch, but if it extends beyond the unit digit, the latter part is treated seperately. If the zero is solely in the unit place, whether or not there is a sequence of fractional zeroes after it, it is simply pronounced 'zen'. This may sound like a complex rule among the many other simple ones used in this system, but it is fairly simple if demonstrated rather than explained in text. A sample number has been given in the image to demonstrate the use of this prefix as well as the other essential points of the dozenite nomenclature mentioned previously. By convention, when zy- combines with doe (one), it is pronounced and written as 'zed' by protocol.

From this point on in this document, any dozenite numbers mentioned in text will use this system of nomenclature so it is recommended that you familiarize yourself with this system before proceeding any further in this document. It is but a child's play to grasp this simple nomenclature system. Nonetheless it needs to be committed to memory like any other nomenclature system for any numeral system in any base.

Note that the page numbers in this document are only of Hindu-arabic decimal format since I have not found a way to include dozenal page numbers via the text editor. That is perhaps the only 'non-dozenal' feature which appears throughout the document. Notwithstanding this limitation, I would recommend that, from this point in the document, the reader use the Dozenite nomenclature for numbers as far as possible, and as the author, I have used this system for denoting numbers from here on in the document (except, of course, for the page numbers and contents page).

Take your time to master this system before proceeding with this document. It will facilitate your understanding of the dozenal system without confusing you with the digits and number names in the decimal system. Basics about arithmetic dealing with multiplcation and division in dozenal (using Dozenite) are to be covered in the following section.

Basic

ARITHMETIC

in dozenal

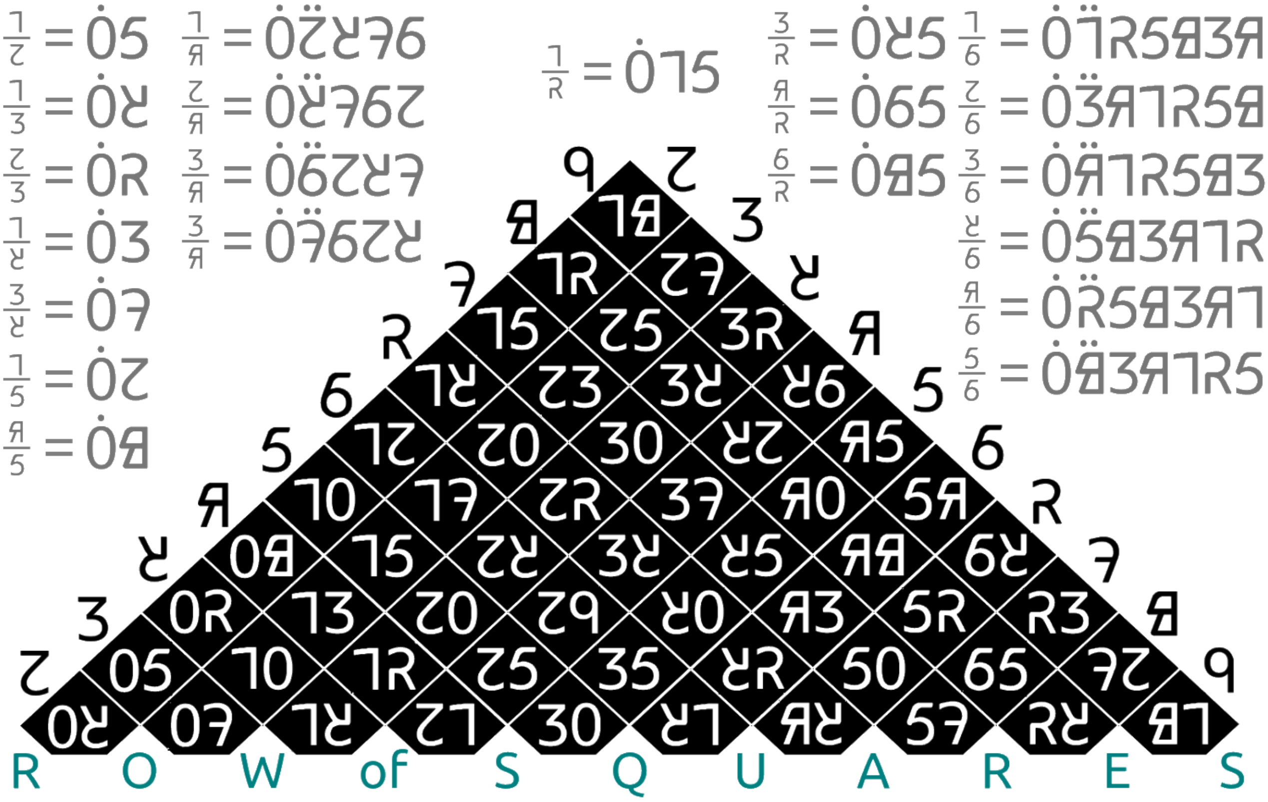
rudimentary tables

Since base twelve is different from the decimal system, even the tables of arithmetic designed on this base are equally different. For those planning on using the dozenal system for arithmetic, it is recommended that the basic multiplication table and fractions listing be taught to the learner. These are rudimentary to the use of arithmetic in dozenal.

This document provides a multiplication table from ren to leph (i.e. two to eleven) and a listing of fractions called fraction table with denominators ranging from ren to echt (two to eight). The digits used from here-on are in the modern Dozenite format. You may refer to these numerals in the previous section for practice. This will act as a springboard for getting used to dozenal arithmetic and its Dozenite representation.

One would have noticed that relatively few among the commonly used fractions have complex or recurring fractional expansions. This is one advantage of the dozenal system. In the Dozenite symbology many results of multiplication seem superficially as if one were using the hindu-arabic numerals in decimal, but don't let this fact get to you as you might get mislead along the way. It can be used as a way to remember the tables easier, but not a steadfast rule.

MULTIPLICATION TABLE



a look at the primes

In dozenal, the prime numbers have a very curious part to play. Prime numbers are positive integers which cannot be divided exactly by any other integer except ‘doe’ (i.e. one). Thus it follows that any positive integer except doe can be expressed as a multiplication of one or more prime numbers. Prime numbers are seemingly orderly numbers which neatly divide into any positive integer, but quite contrarily, no one in the mathematical fraternity has come to a firm conclusion on what pattern the sequence of these prime numbers follow; but that is a topic beyond the scope of this document.

Prime numbers are infinite in number and they do not follow any recognisable pattern in their sequence, so they have remained an enigma to mathematicians so far. The dozenite system does not make any improvement on this front, but it provides a simple way of reckoning with primes. It has been discovered that prime numbers can be categorised into three kinds based on the number four: a factor of four (i.e. ren), a multiple of four minus doe or a multiple of four plus doe; but not all numbers in these categories are prime. This can be and has been proven by simple arithmetic.

In dozenite, we are interested in a more advanced version of this same classification which is based, not on four, but on jeth (six). To spell it out, primes can be categorised into the following categories: a factor of jeth (ren and thir), a multiple of jeth minus doe and a multiple of jeth plus doe. The astute reader, by now, would have correlated this classification with a concept discussed earlier. The first category comprises only two primes: ren and thir, which are the factors of the base number dozen, so they are called base primes. The second and third categories have a curious property. Any number in these categories always has doe, quin, seph or leph as a unit digit. These digits have, as seen earlier, been classified as meridian primes since they are prime and they are obtained by adding or subtracting doe from pole digits zen and jeth. You may refer to ‘the digit cycle’ section for this classification.

The prime nature of doe is questionable but it is considered a prime for now since there is a series of primes which have doe as unit digit. The multi-digit numbers which end in meridian primes are called periodic meridian primes, or per-meridian primes for short. The Dozenite system thus classifies primes as either base primes, meridian primes or per-meridian primes.

In dozenite the tables for multiplication and fractions are not the only ones. There are also prime number tables designed for quick reference. These tables have no pattern by which they can be constructed since they represent a random progression of numbers (the primes). Every block of the table has either an exclamation mark or a cancel to denote prime and non-prime numbers respectively. These tables are basic and cover prime numbers up to a dozyren (a hundred forty four).

Meridian Primes Table

DOZEN / UNIT	0	ꓶ	Ƨ	3	Ꝛ	Я	5	6	Ʀ	Ɂ	ᗺ	9
ꓶ	!	!	X	!	X	!	!	X	!	!	X	X
Я	!	!	!	!	!	X	X	!	!	!	X	!
6	!	!	!	!	X	!	!	X	!	X	!	!
9	!	!	X	!	!	!	!	X	!	X	!	X

	ꓶ	Ƨ	3	Ꝛ	Я	5	6	Ʀ	Ɂ	ᗺ	9
Ꝛn−ꓶ	!	!	!	X	!	!	X	!	X	X	!
Ꝛn+ꓶ	!	X	!	!	X	X	!	X	!	!	X

ꓶ0	ꓶꓶ	ꓶƧ	ꓶ3	ꓶꝚ	ꓶЯ	ꓶ5	ꓶ6	ꓶƦ	ꓶɁ	ꓶᗺ	ꓶ9
!	X	X	!	X	!	!	X	!	!	X	X
X	!	X	!	X	X	!	X	X	X	!	X

Both these tables follow different rules of progression. The table on top is the standard dozenite prime number table which basically lists the per-meridian primes. Each row denotes a meridian prime as the unit digit. Each column is a dozens digit which when combined with a meridian prime unit digit gives a number prime or otherwise. This table doesn't include the base primes since they are assumed to be basic knowledge.

The table below is based on the classification of primes mentioned earlier based on 'four'. This has only two rows for four-n minus doe and four-n plus doe categories of primes. Each column represents the value of 'n' to generate numbers prime or otherwise from the formulae represented by the rows. This table is not a standard since it has only two rows and it covers the list of primes at an excruciatingly slow rate. While the standard table had covered primes up to dozyren with the given dozen columns, this table took doleph columns to cover primes only up to senoe. This table, however, excludes only one prime number, ren, from its listing since that conforms to neither of the two given formulae.

The phenomenon of prime numbers is not unique in different base systems since it is a fundamental mathematical reality which cannot be modified by numeric notation. However, the perspective that one has on prime numbers can vary depending on the base system they use. The dozenal system provides this noteworthy viewpoint on the primes.

rules of divisibility

One reason, as seen earlier, for the mathematician's fascination regarding the dozenal system is the fact that dozen is a highly divisible number. This does not imply that all numbers produce simple divisions when using the dozenal system, but most do. Rules of divisibility in the dozenal system are not hard to remember for most numbers. Note that the notion of a number extends only up to integers when discussing divisibility.

Divisibility basically means the ability of a number (integral) to be divided exactly by a certain factor i.e. without any remainder. The divisibility of a certain number by a certain factor can be ascertained as existent or otherwise by using some tricks of arithmetic. These tricks are summarised as rules of divisibility specified for different factor numbers, and they can be applied on any number (integral). Knowledge of the dozenal digit cycle is a very helpful tool for the testing of divisibility.

Several numbers have straightforward rules of divisibility. They rely only on the knowledge of the units digit. Numbers are divisible by ren (i.e. are even) if the unit digit belongs to the median or antimedian triads. For thir, the units digit must belong to the median quad. For four, the median triad alone, and for jeth, simply the pole digits. These numbers are direct factors of dozen so they are easy to test for divisibility by.

Rules for echt and noe require scrutiny of both the unit digit and the remaining number after removing the unit digit (the dozens digit becomes unit digit of the new number). If there is no dozens digit, it is considered zen. To test for echt, the split unit digit must be zen or echt if the remaining number is divisible by two, and four if otherwise. To test for noe, the unit digit must be zen or noe if the remaining number is divisible by thir; if not, it must be thir when the remaining number is a multiple of thir minus doe, or jeth when the remaining number is a multiple of thir plus doe. This sounds like a mouthful in text, but it is really very simple once you understand it.

Some bigger but composite numbers like dofour, dojeth and doecht also follow similar rules. The following rules test positive for divisibility by these numbers. For dofour, if the remaining number is a multiple of four plus doe, then the unit digit should be four; if a multiple of four plus ren, then echt; and if simply a multiple of four, then zen. For dojeth it is much simpler; if the remaining number is a multiple of thir plus one, then the unit digit should be jeth, and if simply a multiple of thir, then just zen. The rules for doecht are similar to dofour's; if the remaining number is a multiple of quin plus doe, then the unit digit should be echt; if a multiple of quin minus ren, then four; and if simply a multiple of quin, then zen. It will be easier to remember these rules when condensed into reference tables, so they have been included in the appendix section.

So far so good, but matters get more complex when your factor number is a meridian prime.

The meridian primes other than doe require a technique called ‘digit osculation’ for testing of divisibility by them, and this applies even to the per-meridian primes. This is a common technique used in speed arithmetic methodologies like Vedic Mathematics and the Trachtenberg System. It involves a recursive process of splitting the number into unit digit and remaining number as seen earlier, then multiplying the unit digit by a certain integer called an ‘osculator’ before adding it to the remaining number. This process is recursively repeated until only a single digit, negative or positive (or zero), remains. Divisibility is determined by this final digit obtained; to test positive for divisibility it should be (1) the positive or negative of the factor number (2) a positive or negative multiple of that number or (3) just zero. Osculators are specific to the factor number you choose to test divisibility by and there are typically two osculators, one negative and one positive, for each factor number.

The osculators for quin are negative ren and positive thir. Those for seph are negative four and positive thir again. Leph has an osculator of positive doe or negative dozen, which really means you do not require to osculate. There is a shortcut method for leph because its positive osculator happens to be doe. Just adding all its digits to obtain a digit-sum number can be performed recursively until a unit digit is obtained, and it just needs to be leph itself to test positive in divisibility.

Quite the opposite of leph’s osculators are those of dodoe, which are negative doe and positive dozen. However in this case you cannot add all digits recursively and hope to ascertain divisibility thus. However, you can convert every alternate digit into a negative one and proceed thereafter with the kind of shortcut technique given for leph. The next prime on the way, doquin has a whopping negative seph and positive beth as osculators, but they look sizeable in front of doseph’s osculators, negative leph and positive echt. The osculators for doleph, negative dozen and positive ren, are similar to those of leph. These osculators have been listed in a table in the appendix section of this document.

Multiples of these primes by ren or thir can be tested for divisibility by, by simply performing multiple tests on one number. Take a number, which is a multiple of the prime by ren or thir, as a factor number. To test divisibility by this factor number, just see that a number is divisible by **both** the prime and ren/thir (which ever you multiplied by that prime). This technique is, as a matter of fact, obvious. You may do the same by multiplying both ren and thir by the prime to get a factor, in which case you need to ensure divisibility by ren, thir **and** the prime to test positive. Remember, do not multiply the prime by ren or thir more than once. Among such factors, the smallest is beth itself and others include doren, dothir, donoe and dobeth. A mention has been made about them in the appendix section.

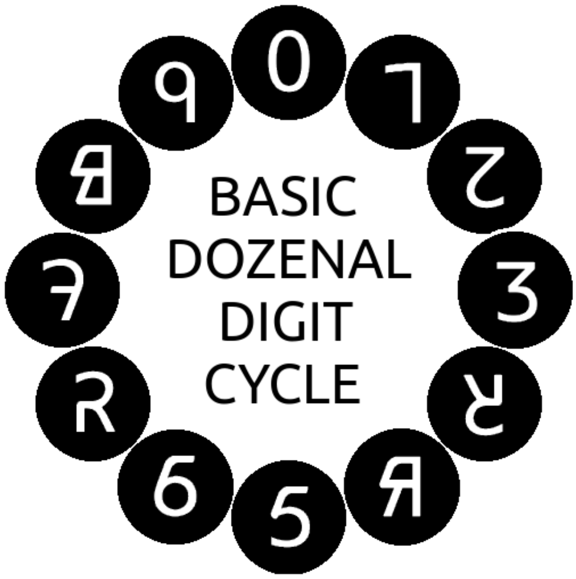
The divisibility rules for numbers from ren to rezen covered in this document provide a firm divisibility knowledge base for a novice in dozenal arithmetic. There are rules of divisibility for all multi-digit factor numbers which use any among the techniques mentioned above with different specifications according to the factor number. It is not hard to figure out these techniques if one can find patterns in multiplication tables. It is, in fact, elementary.

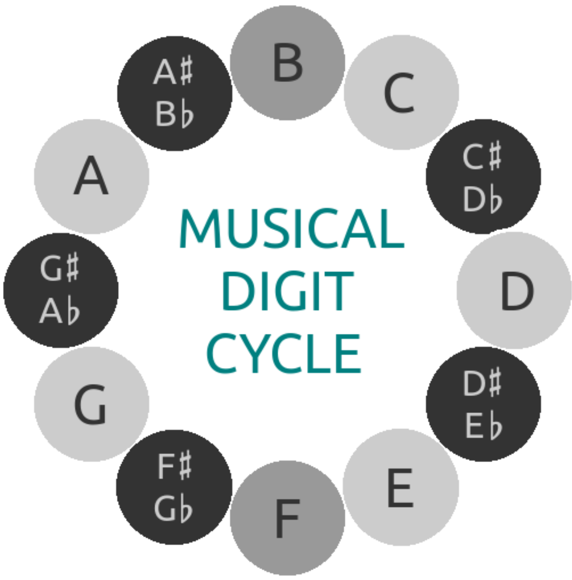
The

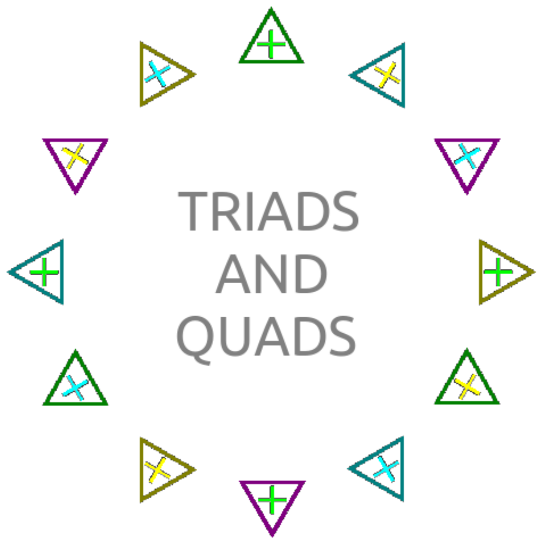
αρρεηδιχ

Section

more on digit cycles







Digit cycles have been discussed briefly a few sections ago along with the idea of triads and quads. The dials shown above have been recognised as standard conventions in the Dozenite system and are fundamental to the dozenite reckoning. The following is a description of these digit cycles and their inter-relationships as observed by the Dozenite system.

The dials shown in the image are pretty direct in their meaning. The top-right one is a basic digit dial using the Dozenite numerals. This is the most primary form of the dozenal digit cycle and can be used as a clock dial, a dozenal protractor (which will be discussed later) or even an old fashioned telephone dial with twelve digits if anyone is keen on designing one.

The top-left dial is a typical colour wheel with the three primary [red, green, blue], three secondary [cyan, magenta, yellow] and six tertiary colour hues [orange, lime, turquoise, sky-blue, mauve, pink] along the dial. This is helpful in gauging how different various hues are from each other just by observing relative positions on the dial and it encompasses a fair number of hues without cutting things too fine (except if you're an artist). If any 2 colours are diametrically opposite on this digit cycle, they are qualified as complementary.

On the bottom-left is a musical note digit cycle which covers all notes and semitones of an octave and is colour coded in shades of gray emulating the black and white keys of a piano keyboard. This is a useful tool for musicians since relative positions of two or more notes on this cycle determine the same harmonic patterns when played out, but on different key notes. For example, every triad is an 'aug' chord and every quad forms a 'dim7' chord. These 2 happen to be poor in harmonicity due to equal intervals, but displacing any 1 note by one digit from these, forms common harmonic chords like major, minor, 7^{th} and minor 6^{th} chords. This provides an aesthetic representation of the notes in any octave where equal angular distance means equal ratio between audio frequency of the notes.

The last dial on the bottom-right basically maps the triads and quads discussed earlier for each position on the dozenal digit cycle. These triads and quads have a property by whose virtue every combination of one triad and one quad maps to a specific position on the digit cycle in a mutually exclusive and exhaustive manner. The positions on the digit cycle are associated with their equivalents in the previous three dials in this reckoning of the dozenal digit cycle. By way of example the number zero, the zero angle, the colour red, the note B and the top position are equivalents of each other and can be pinpointed by the median triad and median quad. It might be helpful to enumerate the different triads and quads so their description has been given below.

By convention a **median** triad and **median** quad are recognised as mentioned in 'the digit cycle' section. These are marked as a green colour triangle and cross-mark respectively on the given dial in the image. There are right-hand medians and quads oriented accordingly and marked in yellow colour. Similarly left-handed ones are oriented accordingly and marked in cyan colour. The three modes of the quad have been covered by these orientations and colouring, but there is a fourth mode of triads. This is the anti-median triad, so called because it is positioned opposite to the median triad. This has been marked as a purple triange oriented accordingly downward. I recommend that you mull over the last couple of paragraphs and carefully observe the image above if you are keen on understanding this.

Several different dozenal digit cycles can be constructed for various purposes but they have not been included here. There is, however, another important use for it as will be seen now.

δozεηιτε ρrοτrαcτοr

A mention of the dozenite protractor has been made a few sections ago in this document. This is a very handy concept that goes hand in hand with the dozenite numerals. The dozenite substitute for a degree angle measure is, in fact, the dozenal digit cycle itself with more detail. The radian measure is not considered standard in dozenite, but it is used for relatively technical work.

Every angle in dozenite is quantified as a fractional expansion between zen and doe, so its notation always begins with a zen in units place. However by convention, it is not pronounced as 'zen' but 'ize-' followed by the fractional digits. The reckoning is simply done on a dozenal clock digit cycle in clockwise direction. Many common angles have simple fractional expansions like 30, 45, 60 and 90 degrees are izedom, izedojem, izerem and izeithem respectively. The 180 degree angle is special since it is associated with negative numbers as will be seen later. While it is generally called 'izejem' it is shortened to 'izm' when denoting negative numbers as will be discussed shortly.

The radian measure of angle is more scientific but less convenient for the layman than the dozenite measure. It is, nonetheless, one that cannot be overlooked, for it deals with a very important number in the mathematical world. This number is a transcendental irrational number known as π (pi), but in dozenite a variant of this number is recognised as a standard for both radian conversion and other mathematical uses. It is known as τ (tau) among dozenalists and had been proposed long before the Dozenite system was even invented. It has the value of ren times π and is deemed by the dozenal community as a more convenient figure to use than π itself. A complete revolution of the circle is τ radians, so what is measured as divisions of 1 in dozenite angle measure is the same as that measured as divisions of τ in radians. In favour of this proportionality between the two systems, the unit of dozenite angle is called a 'tau'.

Like in the metric system, where both time and angle are measured in minutes and seconds, the dozenite protractor measures time also. The system is similar to TGM time measure as proposed decades ago by an ardent dozenalist known as Tom Pendlebury. In dozenite time a day encompasses two phases, ante-meridian and post-meridian (AM and PM) like our current reckoning of time. Each phase is a revolution of a tau. The nomenclature of time doesn't feature 'ize-' since the unit digit can be zen or doe (for AM and PM time respectively). Thus, time measure is greatly simplified and maintains integrity with the radix in use unlike the metric time units. Thus one need simply call 'eight forty-five AM' as 'zen echnom', 'four thirty PM' as 'doe fojem' and twelve midnight as simply zen.

You might wonder why there is a need of coining the unit 'tau' for the dozenite protractor if it measures only angle and time. The fact is that the protractor has another function where it is not measured in tau units, and that relates to **complex numbers**.

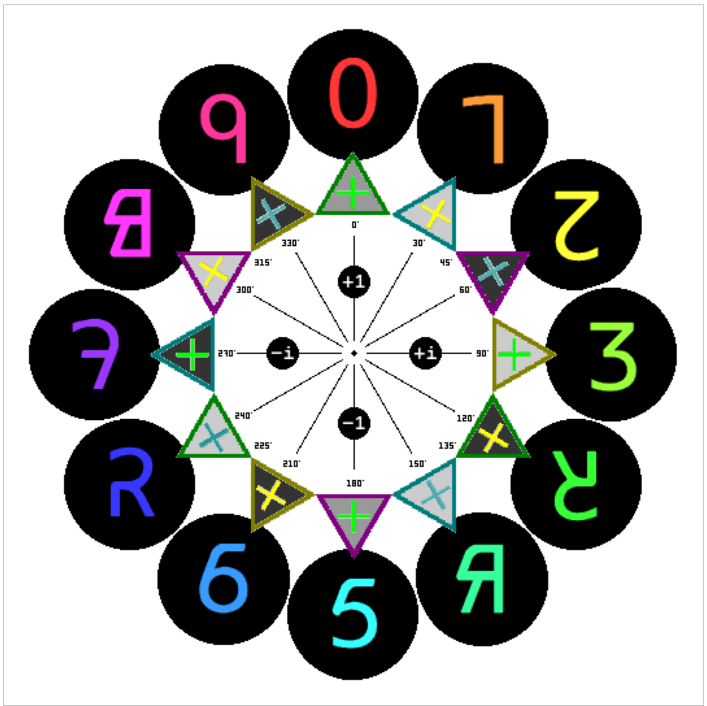
When we juxtapose a dozenite number and a dozenite angle, it transforms into a **complex number** with magnitude of the number and amplitude of the angle. Thus ‘three iota’ can be expressed as ‘thir izeithem’. Note that if you set the complex amplitude to a straight angle of izejem, the number turns into its negative counterpart. For this purpose, the angle ‘izejem’ is contracted into ‘izm’ keeping in mind the frequent use of negative numbers. Thus ‘negative three’ becomes ‘thir-izm’ (or ‘thir izejem’ if you like). It follows via these rules that ‘negative three iota’ is expressed ‘thir izenom’ since it has an amplitude of izenom tau.

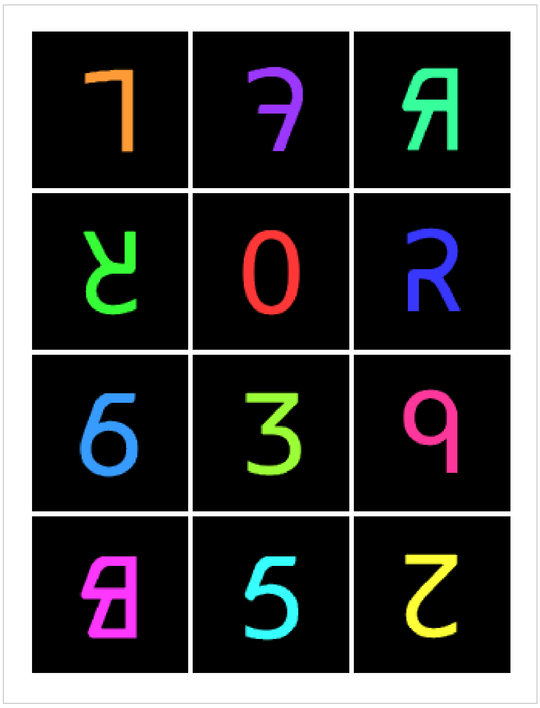
Coming to notation of angles, unlike regular fractional expansions, a circumflex is placed above the unit digit zen instead of a dot. This is often substituted by the ‘Λ’ symbol. When noting time, the doe with circumflex is frequently substituted with the ‘A’ symbol. These notations have been designed to distinguish angles from fractions in the dozenite system.

Have a look at the images below. On the left is a very composite image which describes and correlates the various standard dozenite uses of the dozenal digit cycle. The colour in which each number appears is its corresponding hue on the dozenite colour wheel discussed earlier. The triangles and cancels describe the triads and quads as discussed earlier and the grey backgrounds within each of them represent the musical notes in the musical digit cycle.

The labels of notes have not been given since (1) there are different names for the notes around the globe and (2) note labels are not required anyways. All cyclic dozenite processes occur in a clockwise fashion beginning at the top position. Dark grey positions are for the black notes, medium grey for the pole notes (B & F) and light grey for the other white notes.

Note that the pole notes are distinguished from the other notes because they are the only pair of antipodal notes which are **both** white. With this knowledge, providing the note labels proves to be superfluous and unnecessary.





Apart from these standard digit cycles which were discussed in the previous section, there is the dozenite protractor at the centre of the dial. For the most commonly used angles in each quadrant, the degree equivalents have been given near each digit of the cycle since they also correspond to the standard degree angles of hue in the standard HSV and HCL colour spaces. At each point of the median quad, the complex number value of that amplitude angle has been given too in a small circle as 1, i, -1 and -i. While time measure has not been explicitly portrayed in this image, it would be redundant to show it distinctly since it is the same as angle measure when represented on the dial.

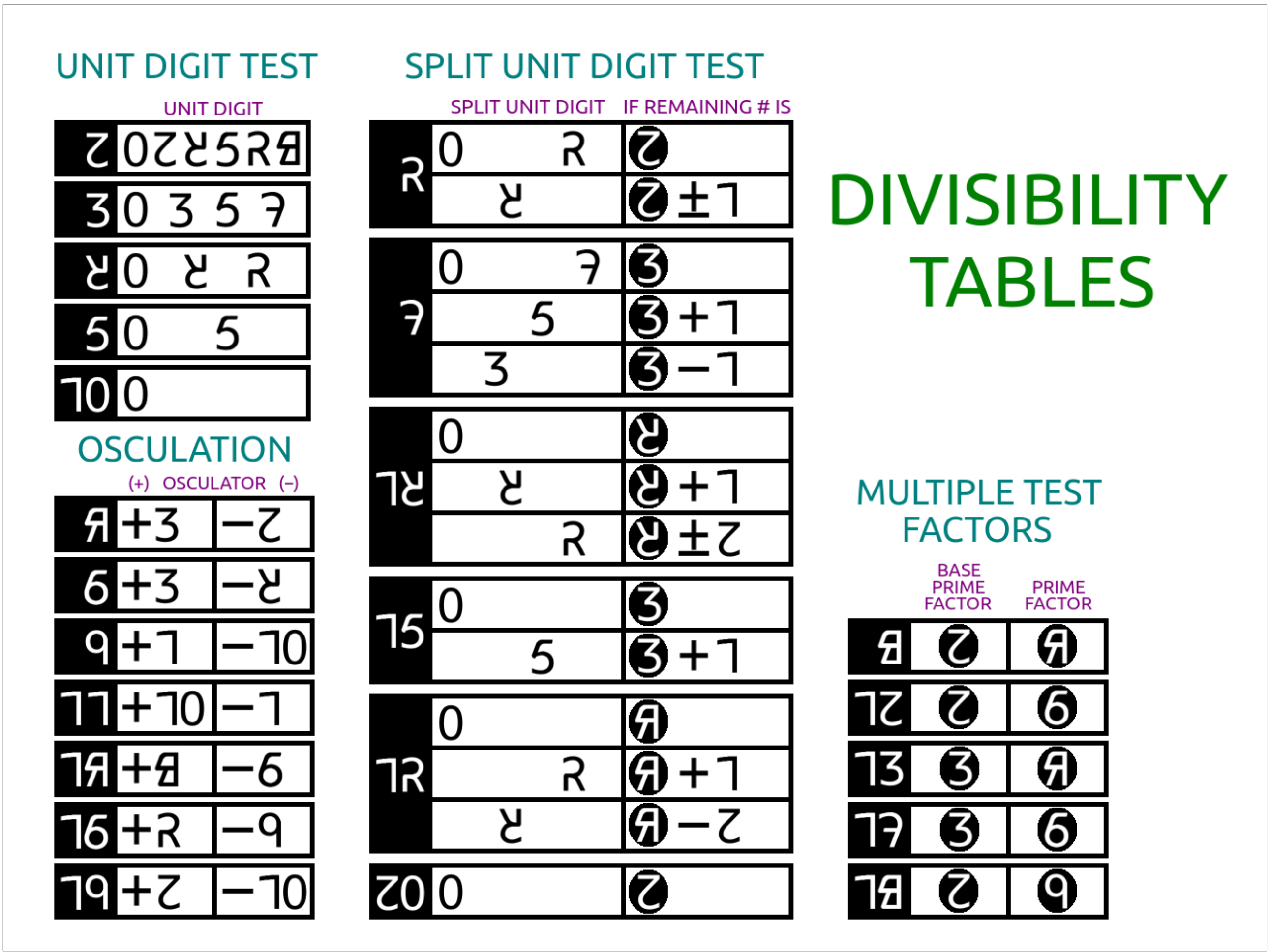
The image on the right is a special variation of the digit cycle called a 'dozenite Tablet'. This has almost no resemblance to a cycle, let alone a digit cycle, but it is useful for demonstrating the concept of triads and quads in a condensed fashion. The digits here are arranged such that every row covers a triad and every column, a quad. The astute reader will also notice that this tablet demonstrates the fact that every digit is specified by a unique combination of triad and quad in an exhaustive manner as mentioned a few sections ago. Of course, this applies not only to the digits but also their correlations in the other digit cycles like the colour wheel or the musical notes' cycle. So one can say that the number quin, the colour turquoise, the note E and the angle izequim (150 degrees) are equivalent and specified by the left-handed triad and quad.

This is the idea in a nutshell, but the digit cycle's influence spreads far and wide. In fact, rather curiously, the dozenite tablet finds itself frequently applied in an apparently remote concept of mathematics, divisibility. Let us revisit the rules of divisibility, now equipped with a better idea of the digit cycle; the dozenite tablet may prove to be useful along the way.

δινιsι6ιlιτγ τα6lεs

As a continuation to the section on divisibility rules, this appendix lists the basic numerical figures to be known before going ahead with a divisibility check. The techniques of divisibility testing are basically the three mentioned previously: (1) unit digit test, (2) split unit digit test and (3) digit osculation, in the order of increasing complexity. The tables below are categorised based on these three techniques and there is a fourth table listing factor numbers which require multiple divisibility tests. This section covers factor numbers up to rezen for divisibility information.

The dozenite tablet mentioned above can provides assistance especially to the adoe (first) and indirectly to the aren (second) technique of divisibility testing mentioned above, for they depict the triads and quads which are used for the adoe technique, so without further ado, let us look at the tables in order to cement our current knowledge of divisibility tests.



It may be helpful to refer to the descriptions of each rule of divisibility given in the 'divisibility rules' section while inspecting this image. Note that whenever a factor number (in this image) is coloured white on a black background, it refers to any number divisible by that factor number, so the top right line in this image would be read out as: a number **divisible by** ren has a unit digit among zen, ren, four, jeth, echt and beth. The line to its immediate right would be read as: a number **divisible by** echt has a split unit digit among zen and echt if the remaining number is **divisible by** ren **or** a split unit digit of four if the remaining number is a multiple of (**divisible by**) ren plus/minus doe. Sounds like a mouthful in text, which is why it has been condensed into an image.

The table given at the bottom right is not a technique of divisibility test in itself. All it says is to test divisibility for such a factor number, just test divisibility for such and such other factor numbers. For example, the first line among these reads: a number divisible by beth is simply divisible by **both** ren (a base prime factor) **and** quin (another prime factor).

This document designed to introduce the Dozenite system of number reckoning, symbology and nomenclature has come to a close. As the author, I would coerce anyone passionate about the dozenal system to consider the content given in the past redoe (25) pages. Thank you.

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δαγ

a document by Punya Pranava Pasumarty