

1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	3	4	5	6	7	8	9	10	11	12	
3	3	3	4	5	6	7	8	9	10	11	12	
4	4	4	4	5	6	7	8	9	10	11	12	
5	5	5										
6	6	6										
7	7											
8	8											
9												
10												
11												
12												



MODULAR COUNTING

A Preview of the Numbers

of the 21st. Century

P. D. Thomas

Revised Edition

PROLOGUE

CIVILIZATION IS BUILT ON COUNTING

For thousands of years, since the fall of Babylon, the progress of civilization has been retarded by the use of inefficient counting systems which have hindered practical mathematics and wasted countless millions of man and woman hours.

We shall never know to what heights of technology the Roman empire might have aspired had they not adopted the most clumsy method of counting in history, with which the basic operations of addition, subtraction, multiplication and division were almost impossible. The Roman craftsmen were as skilled in their own fields as ours today; the Roman organization was in some ways superior to ours; the brains of their thinkers were certainly not inferior.

For many centuries, Roman numerals were the only system of counting known to Europeans. Then, only about 600 years ago, Arabic-Hindu numerals were adopted which, with their zero, allowed mathematics, and technology based on mathematics, to develop. Though a vast improvement on Roman numerals, these symbols, universally used today, were still based on ten - the decimal system - because since the dawn of history, humans had counted on their eight fingers and two thumbs.

In the Space Age, counting on our fingers is no longer good enough. We must grow up and leave the kindergarten. Space Age technologists have already accepted a quiet revolution in counting: they count to a base of two or sixteen as appropriate. Silicon chips have no fingers - they count in twos, so fast that no human fingers can follow them. They cannot be made to count in tens, so decimals have no place in computer technology. Computer operators count in sixteens.

Traders have counted in dozens for sound practical reasons since Roman times. In the 21st. century, everyone can count in dozens if the creaking protests of ignorant, hidebound reactionaries, who never graduated from the kindergarten of finger counting, are overruled and ignored.

These are the heirs of the "scientists" and "experts" who said that heavier-than-air machines could never fly, that radio had no practical use, that space travel was utter bilge - those pronouncements have filled a book.

This is not a technical book; it is intended for the ordinary person who knows that two and two make four and may at some time have wondered why.

It is dedicated to the sixty per cent of the Australian people who find decimal metric more difficult to understand than modular Imperial.

Turn the page and study a blueprint for the next step in the evolution of human progress.

MODULAR COUNTING

A Preview of the Numbers of the 21st. Century

WHY DECIMALS?

Only half a century ago, the idea of counting with any base other than ten (decimal) was unknown outside a select circle of mathematicians - who rarely mentioned it because of a very understandable reluctance to face ridicule from the ignorant. Even educated people apparently could not grasp the principle of "place and point" except in decimal terms, having learnt to count on their fingers.

As recently as August, 1977, Mr. Kenneth Weaver, the Science Editor of "National Geographic" wrote:

"Yet we count in tens, by the decimal system, because 10 is the only number that creates multiples of itself by changing the first digit."

Mathematically, our system of counting is indefensible. A numeral system with a base of a dozen instead of ten units is both logical and desirable, yet the majority of the general public are unaware of its value, and in my experience, cannot understand such a system because it is never discussed. A common remark is: "But tens are easy because you can multiply and divide by moving the point." (So much for education.) A few minutes' patient explanation usually convinces an intelligent person that there is nothing magic about the number ten, since one can have a "decimal point" facility with a base of any convenient number, moving the point right or left to multiply or divide by the base number.

In any case, the advantage of multiplying and dividing by moving a point is largely illusory, since it can only be used when multiplying or dividing by the base. It is of no use at all when multiplying, say, 54 by $1\frac{1}{3}$ (a perfectly simple sum which cannot be worked out in decimals).

THE STORY OF DECIMAL CURRENCY

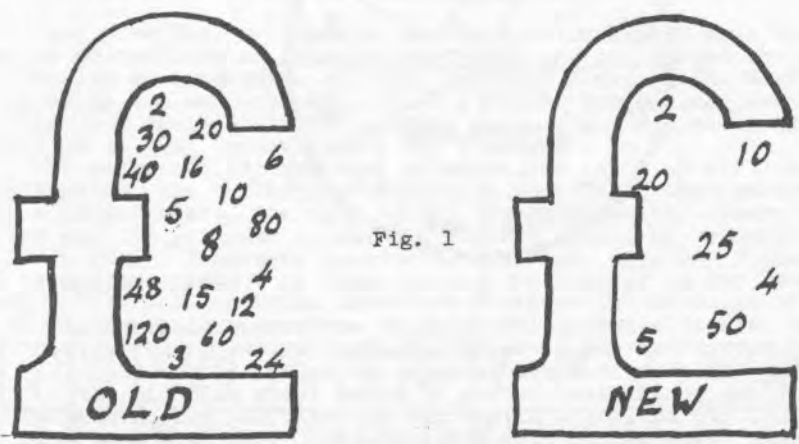
After World War 2, several English-speaking nations (South Africa was one of the first) began seriously to debate the adoption of decimal currency and the metric system - which are two distinct issues, but they were always linked by decimal and metric advocates.

In Britain, the proposal to decimalize the pound Sterling brought a riposte from Professor A.C. Aitken in the form of a book entitled "The Case Against Decimalisation", in which he advocated duodecimal (dozen-based) counting and currency. Subsequently the pound was divided into 100 "new pence", a clumsy and arbitrary system by comparison with the practical division of a shilling into a dozen pence. While amounts above £1 were not affected (as they were in Australia, New Zealand and South Africa), the smaller amounts which everyone handled every day were in confusion. Division of the £, or most round numbers of pence, by 3 or 6 was impossible, and by 4 or even 2 was likely to end up in fractions of pence - so the supremely absurd value of " $\frac{1}{2}$ p" (£0.005) was introduced. This immediately made nonsense of the decimal division, but it was necessary because of the inherent flaw in the decimal system - that 10 has no factors except 2 and 5, and 100 has not many more.

Most round decimal numbers cannot be divided by 3, and there is no advantage in decimal counting unless round decimal numbers such as 10, 100, etc., are used. You can't multiply or divide 365 by 64,

or 6 by 8, by moving a decimal point. In fact "moving the point" can only be done when multiplying or dividing by 10 or a power of ten - something that few people do often in everyday life, whereas we are constantly multiplying and dividing by 2, 3, 4, and the other small factors.

Thus the choice of 100 divisions of the main currency unit, adopted by most countries, is inherently faulty. To be flexible and useful, the small change should be based on a division having the maximum number of factors. The former pound Sterling was divided into 240 pence - a number which is divisible by 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 40, 48, 60, 80, and 120. The new pound Sterling can only be divided by 2, 4, 5, 10, 20, 25, and 50.



MENTAL ARITHMETIC

In the era before inflation, the vast majority of purchases by shoppers were in amounts less than £1 - which were priced in shillings and pence. For centuries, British retailers had calculated most of the amounts of purchases mentally. For instance, a dozen buns at two pence each obviously cost two shillings. 12 pounds of sausages at one shilling and sixpence a pound comes to 18 shillings - one and a half dozen shillings.

Halfpence and farthings, which frequently appeared in prices, were not the problem which they might appear to someone brought up on decimals. If our buns were twopence farthing ($2\frac{1}{4}$ d) each, a dozen of them would be two shillings and threepence, which is $2\frac{1}{4}$ shillings.

Since we counted in decimals, why did we not price the buns at 2.25d and sell a dozen for 2.25 shillings? The reason is simple and adequate. Half a crown, or two shillings and sixpence, could easily have been called 2.5 shillings - but what about two shillings and fourpence ($2\frac{1}{4}$ d)? There is no possible way of expressing that in a decimal fraction except as a recurring decimal. However any number of shillings and pence can be expressed neatly as a common fraction - a penny as $\frac{1}{12}$, 2d as $\frac{1}{6}$, 3d as $\frac{1}{4}$, 4d as $\frac{1}{3}$ and so on. (See Fig. 2.)

In division, if a box of a dozen chocolate bars cost four shillings, the price was obviously 4d each. No conscious thought was necessary for such a calculation. If we make it more difficult and quote a wholesale price of 3/3d for the box, the retailer knows with only a moment's thought (and not a calculator in sight) that he was paying 3 $\frac{1}{4}$ d each for them and by selling at 4d each he would make a profit of $\frac{1}{4}$ d on each.

If the retailer bought 2 oz. bars by the gross carton, he might pay a wholesale price of 17 shillings, which is 17 dozen pence. This is 17d (1/5d) per dozen. By selling at 2d each, he makes a profit of 7d per dozen bars, or 7/- on the gross.

Fig. 2



With decimal currency, calculators are essential items of business equipment. Few people could calculate mentally the cost of three items at 47¢ each in a few seconds, or at all. A pre-decimal counter assistant could give the sum of three items at 3/11d (47 pence) in about two seconds - 11/9d. The old rule for multiplying by a dozen, learnt by schoolchildren for centuries, was: Call your pennies, shillings; and parts of a penny, parts of a shilling. For division, this rule is simply reversed. This is in effect the movement of a "point" to right or left.

THE DECIMAL TRAP

At this stage decimal advocates will say: "That's all very well if you happen to want to multiply or divide by a dozen; but what if you want to use 5, 10, 20, or 100? The rule only holds good for a dozen."

Unfortunately, the said advocates have fallen neatly into the trap. How many times have we heard it said that the decimal/metric system is so simple to use, because to multiply or divide by 10 you just move the decimal point? True - but how often do you multiply or divide by ten? When you do, it is nearly always because things have been artificially arranged in tens to fit into a decimal framework, rather than because ten is an appropriate or a convenient quantity.

They can't have it both ways. If this were a valid argument in favour of decimals, it would be several times more valid as an argument for counting in dozens, since quantities tend to fall naturally into threes, fours, and, yes, dozens; not fives and tens. Few people would buy five of anything unless they happened to be priced at five for, say, a dollar.

In order to utilize the decimal facility, currency had to be divided into units of tens and hundreds, and prices had to become fifths, tenths, etc. of the currency unit instead of convenient quarters, sixths, eighths, etc. since a simple price like 3/2d can never be expressed as a decimal fraction.

In our everyday life we think and act in threes, in fours, in dozens, in quarters - not in fives and tens except where decimal num-

eration requires it. Were this not so, all our units of measurement would have resolved themselves into decimals centuries ago, instead of into dozens and sixteenths, into quarters of this and grosses of that. Thus it is our method of counting that is fundamentally wrong, not our methods of working and measuring.

The persistent usage of the dozen and gross in units of money and measure, and in the packaging and sale of commodities, proves its practical desirability. Who would buy ten eggs? Why are buns, cakes, roses, etc., sold in dozens and half-dozens? Why are small articles, which have to be packed in quantity, shipped in dozens, half-grosses, and grosses?

Let an advocate of decimals try fitting 100 articles, each, say, one inch cube, into packages of different shapes, and then repeat the experiment with a gross. Of course they need not be cubes to prove the point - it is even more enlightening with articles of other proportions, which most merchandise would be.

You can put 100 cubes into a box 4 x 5 x 5 inches, or 2 x 5 x 10 inches, or possibly 2 x 2 x 25 inches. A gross of cubes, however, can be done up into boxes measuring 2 x 4 x 18, 2 x 6 x 12, 2 x 8 x 9, 3 x 3 x 16, 3 x 4 x 12, 3 x 6 x 8, 4 x 4 x 9, or 4 x 6 x 5 inches.

Pick up almost any carton in a supermarket and look at the figures. The majority will be marked "12 x 500g packs", "Contents 24 cans", or some similar quantity.

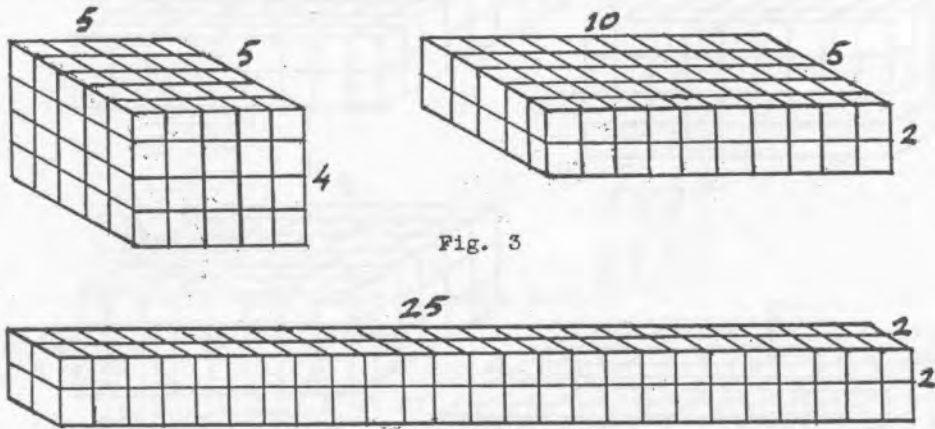


Fig. 3

Fig. 3 shows graphically the only possible ways of packing 100 articles if more than one layer is to be utilized, only two of which are practical in most cases. In Fig. 4 (overleaf) are shown the possible configurations of a gross of the same article, all practical arrangements. Although these are concrete examples of the applications of modular counting, they illustrate vividly the modular principle which also applies to abstract thinking and working.

INTRODUCING MODULAR COUNTING: THE BEST OF BOTH WORLDS

By counting in dozens instead of in tens, we have the best of both worlds - the use of the small modular factors 2, 3, 4, 6, etc., and the ability to multiply and divide by moving the point.

In the past this was usually referred to as duodecimal numeration, but this word is not only cumbersome, it is too easily confused with "decimal". The more modern word "modular" is preferred, and "duodecimal" need only be used in mathematical textbooks or for making direct comparisons between various systems.

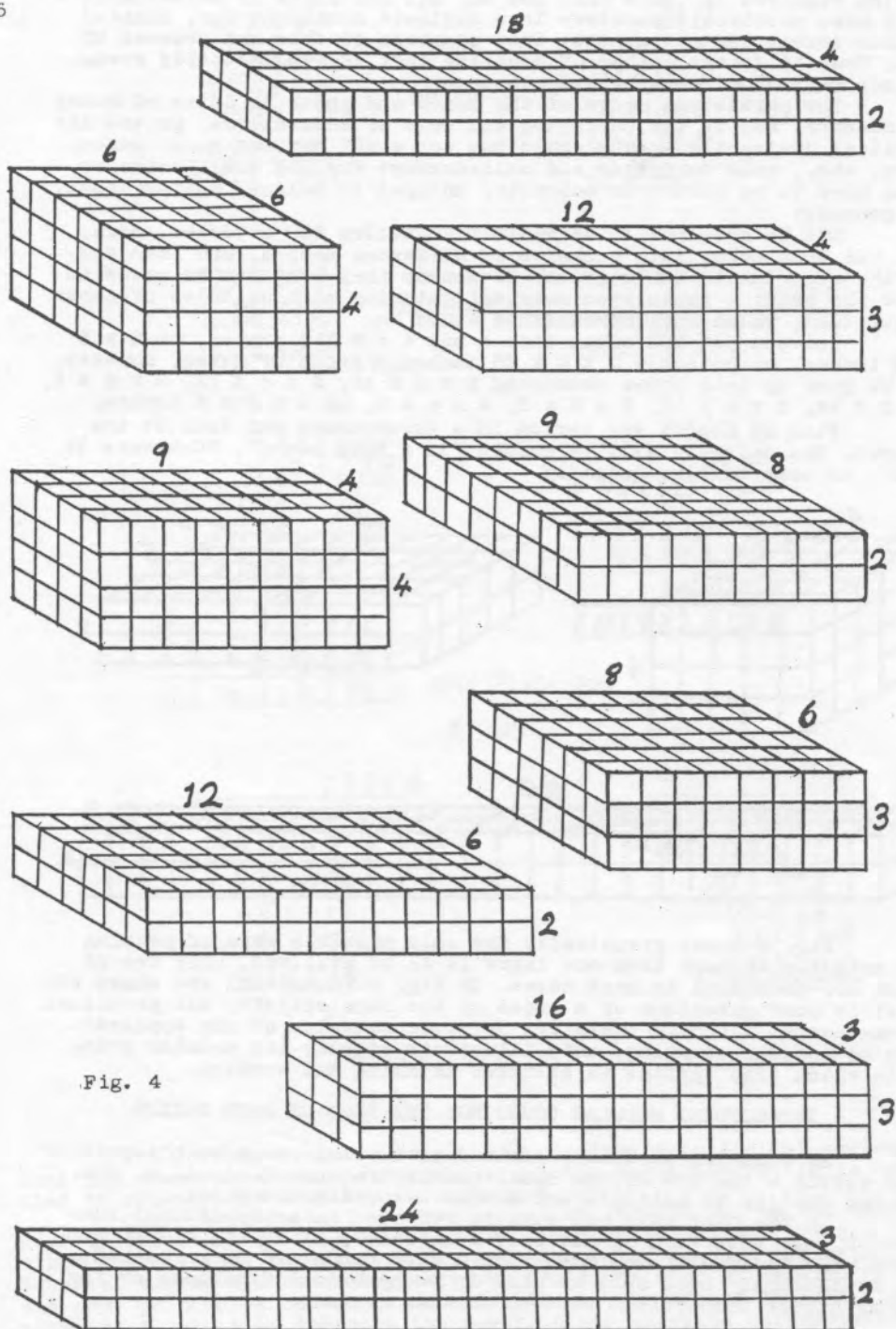


Fig. 4

A modular number, therefore, has the characteristic of being divisible into the maximum number of whole-number components. This drastically reduces the need for awkward fractions in practical use. When necessary, modular fractions, the equivalent of decimal fractions, can be used and for reasonably round figures (half, thirds, quarters, sixths, eighths, ninths, twelfths) require only one or two figures at the point. By comparison, an eighth in decimal fractions requires 3 figures while thirds, sixths, ninths and twelfths cannot be expressed in finite numbers at all.

MODULAR EFFICIENCY

This means that in practical work, modular counting is more efficient, requiring fewer digits and less space and significantly reducing chances of error.

Another advantage in terms of efficiency is that for large numbers, fewer digits are required again. In simple terms, this means that most 5-digit decimal numbers can be written with four modular digits. This may sound like a marginal benefit, but if a telephone dial has twelve figures instead of ten, then using 7-digit numbers, more than three times the number of subscribers can be connected (35,831,807 instead of only 9,999,999). No city would have so many at this time in history, and 6-digit modular numbers could be used in most cities, serving nearly three million subscribers. The saving in time alone by this increase in efficiency is incalculable but is one-seventh of the total dialling time, while the chances of error are reduced by the same amount. Other benefits are obvious.

Professor Aitken has studied the effects and relative efficiency of modular and decimal counting and sums up the situation in these terms:

"In varied and extensive calculations of an ordinary and not unduly complicated kind, carried out over many years, I come to the conclusion that the efficiency of the decimal system might be rated at about 65 or less, if we assign 100 to the duodecimal... If such a waste of time and effort (about 350 hours lost out of every 1,000) were found to be trickling away in any department of a modern production unit, a time-and-work study would at once be set up."

DECIMAL DISADVANTAGES

Professor Aitken poses the question: "Why, in this age of scientific progress, do we endure a system of numeration with so many disadvantages? The answer removes us at once to remote history and probably prehistory; men counted on their fingers, and to this alone, reinforced, it is to be feared, by the indolent, unreflecting, and often arithmetically illiterate force of habit, the survival of the decimal system is due. This cannot however last; men will not always evade decision by the facile and procrastinatory cliché of our times, 'not practicable in the foreseeable future'."

My experience of contact with bureaucrats and politicians has been that they are determined to "evade decision" of what they are too "indolent and unreflecting" to consider. No mathematician, versed in the theory of numbers, would favour the base of ten over the use of a dozen for counting. It is simply ignorance of the theory of numbers that causes politicians and bureaucrats to support the decimal system.

In converting the building industry to metric, it became obvious that in writing specifications, tables, etc. relating to building dimensions, the amount of type required for the decimal metric figures was about double that required for feet and inches. (See the table of window sizes overleaf.) This is the effect of an inflexible system of measurement compounded by inefficient numeration.

WINDOW SIZES

FEET AND INCHES

2'6" x 3"

3' x 4'

4' x 5'

4' x 6'

5' x 8'

MILLIMETRES

750 mm x 900 mm

900 mm x 1200 mm

1200 mm x 1500 mm

1200 mm x 1800 mm

1500 mm x 2400 mm

MODULAR SYMBOLS

One reason why modular counting has not been accepted, or even advocated with real effect, is the fact that no serious attempt was made to provide a set of distinct symbols to distinguish between decimal and modular systems. It has been suggested that two extra symbols should be inserted between 9 and 12, and the Arabic numerals used for the others. This cannot be seriously considered except as a mathematical curiosity. It would take centuries to phase out decimal numerals - perhaps never as long as civilization lasts: for instance the years and the centuries might still be recorded in Arabic numerals, to avoid confusion in chronology, and there is no reason why this should not be done. After 20 centuries, years are still sometimes recorded in Roman numerals without ambiguity!

In computer programming, a mixture of Arabic numerals and letters is used to denote hexadecimal numeration without difficulty, but this is a specialised application operating within clearly defined areas where there is no chance of ambiguity.

Some years ago I devised a series of symbols for modular figures for demonstration purposes and to explore and test the possibilities and advantages of modular counting in practical work. I must stress however that these are not necessarily the best symbols and better ones might be found - provided that they fit the criteria.

They must be distinctive, easily written and recognized, and must not be those already in use for mathematics or other purposes - which rules out Greek letters, for instance, and also letters or figures from other scripts in current use. The symbols, or stylized versions of them, should be capable of being formed by an electronic digital display. This does not have to be the same rectangular figure-8 grid used at present for Arabic numerals.

At the same time the new symbols must have distinctive names, to indicate verbally that they are modular and not decimal numbers. These are the names of the symbols, rather than of the numbers themselves; thus, for small numbers up to nine, decimal names (one, two, three, etc.) can be used interchangeably where these are appropriate or convenient to refer to the numbers which the symbols represent.

The common names for fractions - halves, thirds, quarters, etc. - may be used similarly even for modular fractions instead of saying "point so-and-so".

One important consideration is that the names should be phonetically distinct. "Five" and "nine" are so similar in sound that they can be confused over the telephone, as are "seven" and "eleven". Since we cannot have a dozen separate vowel sounds, and the names should be as short as possible (ideally of three letters), the consonants should be as distinct as possible.

It is useful to have each name begin with a different letter. Apart from aiding the distinctions, this allows the initial letter to be used as an alternative abbreviation.

Instead of each language having its own names for numbers, most of which are unintelligible to the speaker of another language, it is to be hoped that modular names will become international. Some

variations in pronunciation can be expected, but at least they should be as recognizable as such words as "radio", which appears in most European and many other languages.

It is futile to wait for the international community to devise a string of acceptable units so that the possibilities of the modular system can be demonstrated, tested and explained, and the finer points of the system polished by practical experience of trial and error.



A giraffe has been described as an animal which looks as though it was put together by a committee - which is a little unfair to the giraffe, but sums up exactly what happens when committees try to put together works which should be entrusted to wise and experienced individual craftsmen (to use that word in its widest, Kiplingesque sense). Can anyone seriously suggest that the Mona Lisa or "Hamlet" would have been done better by a consortium of international experts?

As a temporary expedient, here are the modular symbols which I have been using for research and demonstration, which will serve the purpose until better or more acceptable ones are devised.

It will be seen that odd numbers include curved lines while even numbers are composed of straight lines.

After several years of use in practical work I have found these symbols to be easy to use after a little practice, and regular use soon commits them to memory.



YUN, Y, = 1, one.

This is essentially the same symbol as the decimal 1, with the addition of a "tail" to distinguish it. The tail is added at the end of the down stroke, pointing to the left. The name "yun" is reminiscent of the English "one" without being close enough to cause confusion. Its value, representing unity, is the same in either system.



DOW, D, = 2, two.

This is a simple zig-zag derived from the mirror image of the figure 2, which can be written very quickly. The name recalls "two" in several languages and has the same initial letter in some of them.



TRI, T, = 3, three.

This is an ancient Runic symbol, but as Runic is now archaic that is hardly likely to cause confusion. The curved arms must be carried right up to the top line, level with the top of the central vertical stroke. The prefix tri- is used in several words (triangle, tripod, trident, etc.) and is a logical name for this symbol.



KAW, K, = 4, four.

The logical symbol for 4 might have been a square, with its 4 sides. However it has some disadvantages, one being that it is symbolic of itself, i.e. a mathematical square. The alternative is a cross, with four arms and four right angles, but this is the universal plus or positive sign. This choice is a cross with a very short horizontal arm, approximately one-third of the height. The name "kaw" is distantly related to "four", "quatre", etc., but is quite distinct.



VIN, V, = 5, five.

The symbol for five units is entirely arbitrary, though it is rather similar to the Greek lambda. However the longer stroke is a

10
single, smooth curve, ending in a perpendicular at the bottom; the shorter stroke being straight, inclined at 45°. The lower part of the symbol therefore makes approximately a right-angled triangle. The initial letter V is the same as the Roman numeral for five.

HEX, H, = 6, six.

The symbol is the capital X with a short horizontal bar like the one in kaw. This gives a symbol with six arms and six angles, easily written and distinguished. The Greek root hex-, meaning six, is a universal prefix for six-sided objects (hexagon, etc.) but the word has never had a currency in its own right in English. It is therefore a logical word to use for the name of one of the most important symbols in modular numeration - six, the half-dozen.

SEF, S, = 7, seven.

The combination of the trident symbol for tri and the horizontal stroke of the symbol for kaw gives this symbol, which is reasonably distinctive, though similar to the astronomical symbol for the planet Neptune. The name "sef" recalls several languages in which the name for 7 begins with the letters SE-, including English.

OKT, O, = 8, eight.

This symbol is developed from kaw; being twice kaw, it has the vertical line with two horizontal arms, spaced about a quarter of the vertical distance from top and bottom. Another common prefix is oct- (October, octagon, etc.) so that is a logical name for the symbol for eight based on the Latin root.

NON, N, = 9, nine.

Like vin, this symbol is arbitrary. It is easily written, with two curved strokes, and is unlikely to be confused with any other symbol commonly used at present. The name is related to "nine" being derived from the same Latin root.

XEN, X = 10, ten.

This is an arbitrary symbol. The name is chosen to rhyme with "ten", to make the equivalent decimal number easy to remember, but it is spelt with an initial X, which is the Roman numeral for ten. It is pronounced "ZEN" like words which have the prefix xen- (from the Greek xenos, "strange"). The horizontal centre stroke does not extend to the left of the slanting upright stroke.

ELF, E, = 11, eleven.

Another arbitrary symbol, a vertical line with a loop in the centre, for the last single digit. The name elf is related to eleven but it has a familiar ring in other languages.

ZERO, Z, = 0, zero.

Zero cannot be anything else and the decimal symbol (0) could have been used without any possible confusion. However, the symbol shown is coming into international use in binary and hexadecimal notation and in telecommunications to avoid confusion with the letter O. Although its use in modular numeration is not essential, it seems reasonable to use it since it is already accepted, and it will be used more and more in the future.

The next step to continue counting from elf is twelve in decimal numeration, or one dozen. The modular symbol for this is made by adding a zero to yun, just as ten is made by adding a zero to one. One dozen and one (13) is yun and yun, one dozen and two (14) is yun and dow, and so on to two dozen (24) which is dow and zero, followed by two dozen and one (25) which is dow and yun, and so on.

The names for the single digits are straightforward and easily handled. However when we come to a dozen and larger numbers, new names are needed. The word "dozen" is well established and unambiguous but rather clumsy. "Yun dozen and yun" has five syllables and would become tedious in use. Modular numbers should be euphonious and lend themselves to rapid counting. They should improve on the larger decimal numbers, most of which have at least three syllables; 77 has five, and 111 has seven.

I have proposed another name for the dozen symbol. This is the word "PAR". It is already used as a reference or base figure - we may speak of being above or below par, in either an abstract or a numerical sense. Since the dozen is the base for modular counting, this is quite appropriate.

Instead of saying "yun dozen and yun", therefore, the next number above par would be "par and yun". This quite naturally becomes abbreviated to "PARA-YUN" in speaking and writing. This is followed by para-dow, para-tri, etc. to para-elf.

The next step is two dozen. Again, "dow dozen" followed by "dow dozen and yun" is too much of a mouthful. "Dow par" is better but "dow par and yun" is not acceptable.

Therefore we return to the simple suffix "-a" which made "par" into "para", and two dozen becomes "DOWA", followed by "DOWA-YUN" and so on to three dozen, which is "TRIA".

These names are set out in the following table.

Decimal Modular		Decimal Modular		Decimal Modular		Decimal Modular	
0	ZERO	12	PAR	24	DOWA	36	TRIA
1	YUN	13	PARA-YUN	25	DOWA-YUN	48	KAWA
2	DOW	14	PARA-DOW	26	DOWA-DOW	60	VINA
3	TRI	15	PARA-TRI	27	DOWA-TRI	72	HEXA
4	KAW	16	PARA-KAW	28	DOWA-KAW	84	SEFA
5	VIN	17	PARA-VIN	29	DOWA-VIN	96	OKTA
6	HEX	18	PARA-HEX	30	DOWA-HEX	108	NONA
7	SEF	19	PARA-SEF	31	DOWA-SEF	120	XENA
8	OKT	20	PARA-OKT	32	DOWA-OKT	132	ELFA
9	NON	21	PARA-NON	33	DOWA-NON	144	YUN GROSS
10	XEN	22	PARA-XEN	34	DOWA-XEN	288	DOW GROSS
11	ELF	23	PARA-ELF	35	DOWA-ELF	432	TRI GROSS

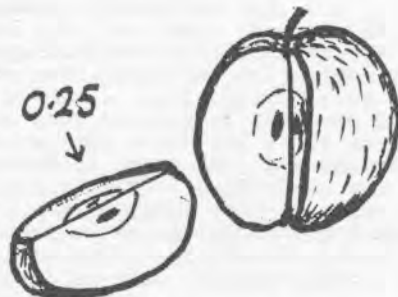
For twelve dozen the name "GROSS" is well established and may be accepted for modular counting, at least for the foreseeable future. Another nomenclature may well be worked out for the larger numbers. For instance, twelve gross or "PAR GROSS" should, to be consistent, have a distinctive, simple name. I shall return to this possibility later.

MODULAR FRACTIONS

Before Napier invented the decimal point in 1617, quantities less than one were expressed in fractions - a half, thirds, quarters, and the rest. These were not easy to calculate, but were easy to understand and visualise. The shape of a quarter apple, for instance, is very distinctive and instantly recognizable, while "0.25 apples" requires some mental effort to translate it into the same slice of fruit.

Fractions have been used since the earliest recorded times - in Genesis we find an offering of a tithe, or tenth.

Napier's brilliant invention of the point and place, allowing multiplication or division by the base (10 in this case), seemed to herald the end of common fractions since any fraction, in theory, could be expressed by the appropriate figures after the decimal point. Unfortunately the inherent defects of the decimal system were shown up in decimal fractions in a way that had never occurred when only whole numbers were used in common fractions, which are simply ratios of whole numbers.



Some simple ratios, such as a half (1:2) or a fifth (1:5) were expressed exactly and quite neatly as decimal fractions - 0.5 for a half and 0.2 for a fifth. But the next simplest fraction - a third - could not be expressed at all except as a recurring decimal: 0.333 to infinity. Another common factor, six, was in the same position; and many of the commonly-used fractions cannot be expressed in decimals.

The result was that centuries later, common fractions are in regular use - they just could not be dispensed with since a calculation involving thirds, for instance, cannot be worked out in any other way in decimal figures.

Apart from fractions, quite simple calculations involving division are often made awkward or impossible, with strings of figures after the decimal point when the result is not a whole number. To take two round numbers at random, 620 divided by 18 is 34.444 to infinity. As a common fraction, this would be $34 \frac{4}{9}$ exactly. If the recurring decimal is rounded to 2 or 3 places (as it must be), and that number is used for further calculations with other inexact figures, the error could be multiplied until it became significant or serious.

MODULAR SOLVES PROBLEMS

Many of these problems are solved by the adoption of modular fractions. Since the base, the dozen, has the factors of 2, 3, 4, and 6, all fractions with those denominators, also twelfths, can be expressed with only one digit after the modular point. In addition, 132 other fractions - eighths, ninths, sixteenths, and so on - can be expressed exactly with only two digits. Thus most of the most often used fractions can be dispensed with, and modular fractions substituted with an economy of figures and an increase in accuracy. Common fractions can still be written in modular symbols; this would be convenient for fifths and tenths, but the use of those would diminish sharply in practical modular working, since they have been used in the past, not because they are inherently useful, but because they fitted into decimal counting.

NAMES FOR MODULAR FRACTIONS

In speaking of modular fractions, it is sufficient to say "point hex", "point zero okt", etc. in the same way as speaking of decimal fractions. However when speaking of common fractions, it is often more convenient to speak of a half, thirds, quarters, etc. for the simpler divisions of a whole. These words are part of the English language, and have exact equivalents in most other languages, and obviously apply precisely to modular fractions as well as decimal.

When speaking of the smaller fractions such as one-twentieth, it would be convenient to have a way of expressing them in modular

terms. The quantity is the same, but if it is expressed in modular symbols, it is appropriate to refer to those symbols by their own names.

Following familiar decimal practice, it is quite acceptable to add the suffix "-eth" for fractions up to $1/12$ (YUN PARETH).

For whole numbers of dozens, the names of which end in "-a" (tria, kawa, etc.), the suffix "-neth" can be used.

This table gives a sample of the names for common modular fractions.

FRACTION	COMMON NAME	MODULAR NAME	MODULAR SYMBOLS
$1/2$	one half	yun doweth	point hex
$1/3$	one third	yun trieth	point kaw
$1/4$	one quarter	yun kaweth	point tri
$1/5$	one fifth	yun vineth	
$1/6$	one sixth	yun hexeth	point dow
$1/7$	one seventh	yun sefeth	
$1/8$	one eighth	yun okteth	point yun hex
$1/9$	one ninth	yun noneth	point yun kaw
$1/10$	one tenth	yun xeneth	
$1/11$	one eleventh	yun elfeth	
$1/12$	one twelfth	yun pareth	point yun

CONVERTING DECIMAL FRACTIONS TO MODULAR FRACTIONS

While whole numbers are converted directly to modular mentally (for small numbers) or by reference to a conversion table, and common fractions by converting the numerator and denominator separately, decimal fractions are more complicated. A decimal fraction is simply a common fraction with a denominator of 10, 100, 1000, etc., depending on the number of places. So it can be written as a common fraction, converted to modular (the conversion is exact except for recurring decimals, which are not exact anyway), and then either divided out to give a modular fraction to the required number of places or left in that form, as may be appropriate.

For example: $0.26 = \frac{26}{100} = \frac{13}{50} = \frac{11}{45} = 0.244$ to three places.

Since all decimal fractions must be tenths, hundredths, thousandths, etc., most of them will become recurring modular fractions. It is ironical that some of the most intractable decimal fractions - 0.16666, 0.33333, and many others - become civilized as soon as they are converted to modular: 0.4 and 0.7 in those two cases. There is a trap in converting such recurring decimals if they are not recognized as simple ratios, since they cannot be converted back to a finite common fraction. For instance, $1/18$ is 0.0555... As a common fraction, this would become (to four places) $\frac{555}{10000}$. Converted to modular, this becomes 0.047 , etc. Yet $1/18$ is 0.055 exactly! The fault here lies with the inherent defect of decimal numeration.

Modular fractions can be converted to decimal fractions if necessary. In this case, the modular figures after the point are converted into Arabic figures as a whole number. This becomes the numerator of a common fraction having 12, 144, 1728, 20736, etc. as the denominator, depending on the number of modular places in the original. This can then be converted to a decimal fraction by dividing the numerator by the denominator after reducing it to its lowest terms.

For example: $0.11X = \frac{11X}{1000} = \frac{162}{1728} = \frac{3}{32} = 0.09375$.

One of the commonest mathematical expressions in daily life and speech is a percentage - sometimes used in a non-mathematical context. Since a percentage is a fraction with 100 as the denominator, that is, a certain number of hundredths, it cannot be used readily to express the simple fractions or ratios except for a half, quarters, fifths, tenths, twentieths, etc.

"Four per cent" means "four per hundred" or four parts in a hundred. The use of a hundred is an arbitrary convention but it shows clearly the faulty nature of decimals. The commonly used percentages are 50%, 25%, 10%, etc. These are simple decimal fractions. But thirds and sixths, which are often used, are intractable and are often expressed as 33% or 66%, which is inaccurate. Eighths need awkward halves ($12\frac{1}{2}\%$) - and so on. As a compromise, whole-number percentages are often chosen, such as 7% or 13%, because they are whole numbers rather than because they represent any significant fraction (who would choose to handle $13/100$ ths in preference to one-eighth?

Taxes, dividends and other applications of percentages are often based on such fractions of the principal as $1/16$ ($6\frac{1}{4}\%$), $1/12$ ($8\frac{1}{3}\%$), $1/8$ ($12\frac{1}{2}\%$), or $1/6$ ($16\frac{2}{3}\%$). Such fractions of a gross are of course all whole numbers, as well as $1/36$, $1/24$, $1/18$, $1/9$ and all the range of numerators of such fractions.

With modular counting, while percentages can still be used if convenient, the preferred expression is a "PERGROSSAGE" - a fraction expressed in $1/144$ ths. The number of simple ratios which can be expressed as whole-number pergrossages is immediately expanded, and many more can be expressed exactly with an extra modular place or two. At the same time, more whole numbers are available in pergrossages - nearly half as many again as the available whole number percentages - which makes fractions less necessary. Not only is the system tidier, it is much more flexible.

Therefore the general adoption of the pergrossage system to supplement or replace percentages is long overdue. It only required a set of modular symbols and names to be practical.

The % symbol could still be used, but to emphasise that the number is a fraction of a gross, two small modular zeros ($\emptyset$) could be placed one above the other with a short horizontal stroke linking the oblique strokes.

PRACTICAL APPLICATIONS

I first experimented with modular counting in the 1940's, but found the lack of suitable symbols was a handicap to practical use. After devising the range of symbols shown here, I made use of them in a public lecture in the 1960's as a demonstration of the use of modular numeration. In the previous 20 years I had several times written letters to the Press, advocating the adoption of a modular system, and received some support - and also some abuse from ignorant or hide-bound advocates of decimalization. A fairly characteristic criticism came from a contributor who hid behind the nom de plume "Decimus": "Under a duodecimal system of notation the figures 21 would cease to mean twenty-one, but would mean twenty-five; and 36 would mean forty-two. Imagine trying to introduce a change like that! Only the profoundest mathematical brain would stand the strain..." and so on.

All that rhetoric was of course completely unnecessary since Arabic numerals would not be used in a modular framework. It would not be necessary to change a single decimal figure or scrap a page of text since there would be no ambiguity.

All that is required of anybody is to memorise the modular symbols; familiarity comes quickly with practice. The introduction of modular counting into the school curriculum from the earliest years would produce in ten or twelve years a generation which is completely

familiar with modular use, counting in dozens in preference to tens because of its manifestly greater flexibility, but still familiar with decimal numeration.

Using the modular symbols shown here, I have carried out practical work and measurements and found that the expected advantages are in fact realized. There is naturally no advantage in using metric measurements, which are bound for all time in a decimal strait jacket; but in using feet and inches, square feet and square inches, cubic feet and cubic inches, the flexibility is astonishing - anyone who is not accustomed to using the simple factors and fractions might be bewildered by the possibilities which open up.

To take a very simple example, a piece of material or a measured length which is sixteen inches long could be expressed as $\downarrow\downarrow$ in. or $\downarrow\downarrow$ ft. A square of that size could be $(\downarrow\downarrow \times \downarrow\downarrow)$ sq. in. = $\downarrow\downarrow\downarrow$ sq. in. or by inserting a modular point (dividing by a gross) $\downarrow\downarrow\downarrow$ sq. ft.

The capacity of a refrigerator, which in metric is clumsily expressed in litres and requires tedious calculation to derive from its dimensions in mm, is easily found with modular measurement.

To find the capacity in cubic feet of a refrigerator which measures 28" x 18" x 22": First find the capacity in cubic inches by multiplying these dimensions.

DECIMAL

$$\begin{array}{r} 28 \\ 18 \\ \hline 28 \\ 224 \\ \hline 504 \\ 22 \\ \hline 1008 \\ 1008 \\ \hline 11088 \end{array}$$

MODULAR

$$\begin{array}{r} \downarrow\downarrow \\ \downarrow\downarrow \\ \hline \downarrow\downarrow \\ \downarrow\downarrow\downarrow \\ \hline \downarrow\downarrow\downarrow \\ \downarrow\downarrow\downarrow \\ \hline \downarrow\downarrow\downarrow \\ \downarrow\downarrow\downarrow \\ \hline \downarrow\downarrow\downarrow\downarrow \\ \downarrow\downarrow\downarrow\downarrow \\ \hline \downarrow\downarrow\downarrow\downarrow\downarrow \end{array}$$

Now reduce cubic inches to cubic feet.

$$11,088 \text{ cu. in.} = 11,088 \div 1728 \text{ cu. ft.}$$

$$\downarrow\downarrow\downarrow\downarrow\downarrow \text{ cu. in.}$$

$$\begin{array}{r} 64166 \text{ cu. ft.} \\ 1728 \overline{) 11088.0000} \end{array}$$

$$= \downarrow\downarrow \cdot \downarrow\downarrow\downarrow \text{ cu. ft.}$$

$$\begin{array}{r} 10368 \\ 7200 \\ \hline 6912 \\ 2880 \\ \hline 1728 \\ 11520 \\ 10368 \\ \hline 11520 \\ \text{ETC.} \end{array}$$

It requires no imagination to see which is the more efficient system. The initial multiplication is slightly simpler in modular than in decimal figures, and the product has only four digits instead of five. When it comes to converting cubic inches to cubic feet or vice versa, it is only necessary to move the modular point three places instead of working a long, and usually indeterminate, division sum.

The following examples of addition, subtraction, multiplication and division show the relative simplicity of modular arithmetic. In each example the same numbers are used, expressed in decimal notation on the left and modular notation on the right. In most cases the modular working is simpler and shorter than decimal working.

$$\begin{array}{r}
 1581 \\
 13854 \\
 454076 \\
 88360 \\
 \hline
 658504 \\
 \hline
 1216375
 \end{array}$$

ADDITION

$$\begin{array}{r}
 \text{L P U} \\
 + \text{P S X} \\
 10 \text{L P U} + \\
 + \text{P P S} \\
 \hline
 4 \text{P U P U} \\
 \hline
 + \text{L P P U}
 \end{array}$$

$$\begin{array}{r}
 227038 \\
 88360 \\
 \hline
 138678
 \end{array}$$

SUBTRACTION

$$\begin{array}{r}
 \text{L P P L} \\
 + \text{P P S} \\
 \hline
 \text{X P P U}
 \end{array}$$

$$\begin{array}{r}
 329252 \\
 138678 \\
 \hline
 329252 \\
 987756 \\
 2734016 \\
 1975512 \\
 2304764 \\
 2734016 \\
 \hline
 45760108856
 \end{array}$$

MULTIPLICATION

$$\begin{array}{r}
 \text{P P L X X X} \\
 \hline
 \text{X P P U} \\
 \hline
 \text{P P S P U} \\
 \hline
 \text{L P P U} \\
 \hline
 \text{P P P P X} \\
 \hline
 \text{L P P U} \\
 \hline
 \text{P P P P U} \\
 \hline
 \text{P P P P U}
 \end{array}$$

$$\begin{array}{r}
 13854 \\
 227038 \overline{) 3145384452} \\
 \underline{227038} \\
 875004 \\
 \underline{681114} \\
 1938904 \\
 \underline{1816304} \\
 1226005 \\
 \underline{1135190} \\
 908152 \\
 \underline{908152}
 \end{array}$$

DIVISION

$$\begin{array}{r}
 \text{P S X} \\
 \text{L P L} \overline{) \text{P P P P P P P P}} \\
 \underline{\text{P P P P P}} \\
 \text{P P P P P} \\
 \hline
 4 \text{P P P P} \\
 \hline
 10 \text{L P U} \\
 \hline
 22 \text{P P P} \\
 \hline
 22 \text{P P P}
 \end{array}$$

The working is the same as the procedure with decimal numeration. The same rules apply and where a modular point occurs, it is handled in exactly the same way as a decimal point.

Other mathematical operations can be carried out in the same way. Calculations of interest, problems in proportion, and all the other arithmetical exercises follow the same rules as with decimals. Squares, cubes, square roots, and cube roots can also be handled on the same principles. There is no objection to the use of decimal figures (2, 3, etc.) to indicate the power. Other accepted mathematical

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Geometry and technical drawing benefit from modular numeration. The circle, divided into 360 degrees (30 dozen, or $2\frac{1}{2}$ gross) is already a modular-based entity. No one would attempt to divide a circle into 100 degrees - although there was an attempt to foster a 400° circle, which was an utter failure because of its impracticality. But then, decimal devotees are not practical people.

For practical arithmetic, multiplication tables have been memorised by generations of schoolchildren and are not always the most-loved aspects of scholastic studies. Marjorie Fleming, the Scottish child prodigy who corresponded with Sir Walter Scott, once wrote to him: "The most devilish thing is eight times eight and seven times seven, and it is what Nature itself can't endure".

It is not necessary however to memorise, say, 3×7 and also 7×3 , since the product of those or any two figures is the same. The logical multiplication table therefore contains the products of any pair of figures once only. Here is the modular multiplication table.

[illegible]

It would be ideal to have a coherent system for naming large and small numbers and fractions, with a series of prefixes for measurement units, derived from them and therefore instantly recognizable. This has never been done in the history of numeration and measurement.

We speak of a thousand or one-thousandth, yet the metric prefixes for the corresponding units are "kilo" and "milli". The only example of coherent usage which gained acceptance is the word "thou", used in engineering to mean a thousandth of an inch.

SQUARE AND VOLUME MEASUREMENT

Square measurement would follow the same ranges as linear measures, based on the square foot and square inch (for which distinct names could be found) using the appropriate prefixes. The intermediate ranges of 3" and 4" squares and their multiples and submultiples are also available. The square yard has been a valuable unit where the square foot is too small and a larger unit, the acre or square mile, is too big. It fits perfectly into a modular system in an intermediate range.

Volume would be based on cubic inches and cubic feet. Again intermediate units are instantly available - a 4" cube, which is close to a quart or a litre; or a 3" cube if that is too big. These divide readily into the primary unit of a cubic foot.

FOUNDING A COHERENT SYSTEM OF MEASUREMENT

It is now possible to construct a completely coherent system of measurement based on the foot/inch standard, completely modular in its relationships, which will integrate units of measurement of all physical quantities. The only unit which does not at present fit this system is the unit of mass - neither the pound nor the kilogram will serve exactly, and an approximation is not good enough. That would cause the same type of flaw that at present gives a relationship between the newton and the kilogram of slightly less than ten.

A new unit of mass can therefore be adopted which will serve the purpose without compromise or confusion.

This coherent modular system is at present being drafted and will soon be ready for publication.

TEMPERATURE

The Fahrenheit scale is modular, since there are 180 degrees between the freezing and boiling points of water, adjusted to give a practical scale of air temperatures. It is more logical and efficient than the decimal centigrade scale of water temperatures, which requires negative quantities and decimal places to be used for atmospheric temperatures. Since we live in air and not water, it is absurd to use centigrade for our environment.

TASKS FOR THE FUTURE

Before modular numeration can become universal, modern technology must come to terms with it. An acceptable type face for the symbols must be settled to allow their use in the print medium; this is a minor step, though an important one. As most figure manipulation is now done electronically, a suitable readout and printout form of the symbols will have to be devised.

Typewriters having interchangeable typefaces (such as "golf balls") will readily accept modular symbols.

The next step is to produce pocket and desk calculators which will convert decimal to modular, and vice versa, and which will carry out all the required mathematical functions in either system. This presents no problems.

There is a bonus with a modular calculator. The ordinary pocket calculator can only accommodate so many digits - perhaps 8. This means that only numbers less than a hundred million can be handled, or a decimal fraction to 7 places.

However a modular calculator with 8 digits can handle numbers up to 429,981,626 (the 7th. power of 12). This multiplies its capacity by more than 4 times. With only 7 digits, it could still handle nearly 3 times the highest decimal number.

Since computers and calculators require all quantities less than unity to be expressed at present in decimal and not common fractions, modular numeration would have yet another advantage in that more commonly-used fractions can be simply expressed as modular than as decimal fractions.

It is probable that computers of the future will operate directly in modular by the use of a tristable (three-state) device, which is already theoretically possible, in combination with the bistable (two-state) devices used at present. The latter can only operate with binary numeration, which is suitable for the internal working of computations, but quite impractical for the finished readout. The modular capability would eliminate the need to use hexadecimal (16-based) numeration for writing computer programmes.

LESSONS OF HISTORY

Perhaps the most momentous decision in the history of measurement was made after the French revolution when the academicians were discussing their new metric system. It was proposed that it should be based on modular counting - the great scientist Pierre Laplace was one of the advocates. This was vetoed however by other academics who said that the French people would not be able to understand it! Other famous advocates of modular were Blaise Pascal, the 17th. century mathematician whose name, ironically, has been given to the SI metric unit for pressure; and Gottfried Leibnitz, who invented the calculus in the 17th. Century.

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