THE CASE AGAINST DECIMALISATION

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Introduction

It has long been known to mathematicians that the system of numeration which, by gradual evolution, we have inherited from previous ages and now use, namely the decimal system, is not the ideal system. Equally it has been known that there has always existed a superior system, the duodecimal, certainly possessing some defects — since no system can be perfect — but superior in all important respects to the decimal system. The great names in the list of those who have explicitly criticised the decimal and upheld the duodecimal are: Blaise Pascal, that outstanding mathematical and religious genius of the early seventeenth century; Gottfried Wilhelm Leibniz, philosopher and theologian, joint inventor with Newton of the differential calculus, first of all names in perceiving the possibility of expressing logic itself in mathematical terms and notation; Pierre Simon Laplace, the celebrated mathematician of the later eighteenth and early nineteenth century, expositor of celestial mechanics, founder of the modern mathematical theory of probability, a name still associated with formulae and methods which are household words in mathematical analysis. Pascal in 1642 at the age of nineteen invented an adding machine; Leibniz in 1673 at the age of twenty-seven exhibited an adding-and-multiplying machine at the Royal Society of London.

As for Laplace, he is related to our topic by the fact that with Borda, Condorcet, Lagrange and Monge he was one of the Commission set up by the French Academy of Sciences in 1790 to examine the possibility of a decimal system of metric and of currency, and to take steps to introduce it. It is known that in the early stages of these deliberations the possibility of a duodecimal system, recognised as superior to the decimal, was discussed; but that it was rejected, on the ground that it was out of the question to educate the French public, within reasonable time, in this kind of calculation. In Britain, where the dozen had more uses, these considerations might have weighed less. At any rate decimal currency was imposed on France in 1795, and the metric system, which ought logically to have preceded or been simultaneous with the currency change, since commodity and its measurement logically precede the monetary medium, was postponed until 1799. This however was not intentional; both changes would have been made together but that the quadrant of the Earth had to be accurately surveyed (as was done by measuring an arc between Dunkirk and Barcelona), and this difficult piece of geodesy could not be completed before 1799. Only then could the standard metre be adopted.

When all was over, regrets were felt by some, not then but later. Laplace himself in his later years gave expression to these; and one can hardly doubt that when, in his last recorded words as uttered to his disciple Poisson, "L'homme ne poursuit que des chimères", he included, among those phantoms captured and found wanting, the decimal and metric system. Napoleon himself (Napoleon's remark was characteristic: "Twelve as a dividend has always been preferred to ten. I can understand the twelfth part of an inch, but not the thousandth part of a metre") expressed regret for the extirpation of the number twelve from numeration and from exchange, for that is what any proposal of wholesale decimalism implies. It implies indeed, as will be shown in cumulative detail later in this essay, the elevation to an undeserved place of a very unsuitable integer, namely ten, whose only distinctive property is that it divides by five, with the consequent demotion of twelve, a number divisible by 2, 3, 4 and

6, while its square, the gross, 144, divides by these and in addition by 8, 9, 12, 16, 18, 24, 36, 48 and 72, with all the consequences of economical and suitable use in parcelling, packaging, geometrical and physical construction, trigonometry and the rest, to which any applied mathematician and for that matter any practical man, carpenter, grocer, joiner, packer could bear witness.

Once again, currency should come afterwards and subserve all these; it should be in a one-to-one correspondence with them, which is indeed the reason for the traditional and wellgrounded British preference for the shilling with twelve pence, the foot with twelve inches; and also for the relation of the foot to the yard, since the number three, so intractable in the decimal system (consider one-third, 0.33333 . . ., or the similar equivalents for a sixth, a twelfth and the rest), precedes the number five in order, use and logic. The twenty shillings to the pound was a characteristically British (indeed not British but English) attempt at reconciliation and compromise, for the French used not so much ten as the score (e.g. quatrevingts, quatre-vingt-dix), and this accommodation of twenty as well as twelve produced our hybrid system of pounds, shillings and pence, the disadvantage of which is precisely that it *is* hybrid, and therefore does not lend itself, as the decimal system does, to a "place" and "point" system of numeration. (A suggestion for rectifying this defect will be given later in this essay.)

With all this, however, pounds and pence have an advantage which the franc and centime, dollar and cent, metre and centimetre, cannot possibly claim, namely the exceptional divisibility of the number 240. This in fact is one of those integers which mathematicians, in that special field called the "theory of numbers", are accustomed to call "abundant". An abundant number is one that has more factors than any number less than it; other examples of small size are 12, 24, 36, 60,120, 360. The gross, 144, or twelve dozen, just misses abundancy, being excelled by 120. Compared with 120 and 144, even with 60, the number 100 is relatively poverty-stricken in this respect — which indeed is why the metric system is a notably inferior one; it cannot even express exactly for example the division of the unit, of currency, metrical or whatever, by so simple, ubiquitous and constantly useful a number as three.

We are therefore entitled to ask: why, in this age of scientific progress, do we endure a system of numeration with so many disadvantages? The answer removes us at once to remote history and probably prehistory; men counted on their fingers, and to this alone, reinforced, it is to be feared, by the indolent, unreflecting, and often arithmetically illiterate force of habit, the survival of the decimal system is due. This cannot however last; men will not always evade decision by the facile and procrastinatory cliché of our times, "not practicable in the foreseeable future". In later paragraphs it will be indicated how new kinds of electronic computers, and the new type of education that this will enforce in the schools, universities and colleges of technology, are bound to produce a full acquaintance with four systems of numeration at least: (i) the binary, based on two, the foundation of all electronic computation, to the exclusion (meanwhile) of the decimal except at the final stage of conversion and recording results; (ii) the octonary, the system based on eight, by which binary results may by the simplest of transformations be compressed and held in store; (iii) the decimal, since unfortunately, with all its defects, it is still with us; (iv) the duodecimal, which in the opinion of many such as the writer will prove to be that system which translates the binary to the world at large, the world of men and women behind counters, ticket offices, carpenters' benches, in stores, in homes.

History of Numeration

With such various introductory remarks, let us look at the history of numeration. We know of course, arithmetic in primitive times being necessarily primitive, that counting and barter were done on the fingers (whence the name digit for a number-sign), and that these hardened into written marks or into such movable objects as the beads or counters on the Chinese, Japanese or Russian abacus. On the abacus, for example, the several parallel rods carrying counters are all crossed at right angles halfway along by a fixed dividing bar; each rod has on one side of the bar five counters, on the other side a single counter. (The number five, it is interesting to note, can be represented in *two* different ways; either, with the thumb, push all five counters up against the bar, or leave them alone and with the finger pull that other counter back against the bar.) The abacus, used by an expert, has remarkable resource and speed; during the American occupation of Japan, a Japanese with an abacus beat an American using a hand-operated calculating machine. The whole point of mentioning this here is that if, for example, Russia should ever go duodecimal, a not unlikely possibility which would give her people, in all the ordinary calculations of life, an advantage of at least 35 manhours—so I reckon—in every 100, China could align herself with Russia even more simply, by having six counters *instead of five* on the half-rod of every abacus.

Ancient History

But to return to ancient history. The Sumerians of two thousand B.C., as is shown by certain cuneiform inscriptions brought to light not so long ago, used the ten system but also the sixty, the sexagesimal system; we have for example their multiplication tables. By 1800-1700 B.C. something quite extraordinary takes place; the Babylonians take over from the Sumerians, and while still in the market place the scale of ten persists, the astronomers, architects, in fact what one may call the mathematicians, scientists, technologists of that remote period, the Hammurabi dynasty of 3700 years ago, constitute a hierarchy skilled in arithmetic to a degree unrivalled in the modern world; for they actually used the scale of 60, the sexagesimal scale, for fractions, reciprocals, even square roots. They have left the trace of their system in the 60-fold division of the hour into minutes and the minutes into seconds, a predominantly duodecimal subdivision, as one may see by looking at a clock, but in this we observe an accommodation not so much with the scale of ten as with 5.

Another such trace is the division of the whole circumference of the circle into 360 degrees. At the time of the French Revolution certain fanatical decimalists (following in the footsteps of Stevinus of Bruges two hundred years earlier) were for dividing the right angle into 100 degrees called "grades", the half day into ten hours, even the year into ten months. These efforts, or rather the second of them, met with no success. Astronomers and surveyors will never use so defective a system; and numbers of instances can be cited, from trigonometry, periodic analysis, approximate evaluation of areas and volumes, and so on, in which a five-fold or ten-fold subdivision of the range gives formulae and methods remarkably inferior to a six-fold or twelve-fold one. Those Babylonian mathematicians, by the way, have extensive tables, not only of reciprocals and square roots but actually of triads of integers making the sides of a right-angled triangle, the theorem of Pythagoras 1150 years before Pythagoras; but all in sexagesimal. The central point in all this is that 60 is an "abundant" number. That was why the Babylonians, masters of arithmetic in a way that, with certain exceptions, we are not, used it as a suitable base for their numerical system.

The Egyptians were not good at arithmetic; they could " do sums", but even the addition of vulgar fractions was carried out by them in an unbelievably cumbrous manner. The Greek system of numeration was an inconvenient one, letters of the alphabet being used for numbers. The Roman was hardly better, except that with a special kind of abacus they used a duodecimal notation for fractions, traces of which survive in two of our nouns, ounce and quincunx, that is to say, a twelfth and five-twelfths. For integers, however, they used the ten system and their well-known numerals; beautiful (none better, said Eric Gill) for lapidary inscriptions and coins, of no use for convenient calculation. These endured in arithmetic almost up to A.D. 1500, simply because of the all-pervading dominance of the Roman Empire, and later of Rome itself. In Asia this was not so; Hindu arithmetic had evolved special single symbols for the integers up to nine, together with the zero, long believed to be a Hindu invention until lately rediscovered, in an analogous role, in Babylonian cuneiform. This Hindu system, with its excellent "place,' convention, though not yet extended to fractional use with the "point", percolated to Europe by way of the Arabs (for what we call Arabic numerals ought more justly to he called Hindu-Arabic), and the geography, early steps and manner of this percolation are worth a brief interlude.

Here it is convenient for speed to link in sequence a few sentences from Cajori's *History of Mathematics*: ". . . at the beginning of the thirteenth century the talent and activity of one man was sufficient to assign to the mathematical science a home in Italy.... This man, Leonardo of Pisa, ... also called Fibonacci, . . was a layman who found time for scientific study. His father, secretary at one of the numerous factories on the south and east coast of the Mediterranean erected by the enterprising merchants of Pisa, made Leonardo, when a boy, learn the use of the abacus. During extensive travels in Egypt, Syria, Greece and Sicily ... of all methods of calculation he found the Hindu to be unquestionably the best. Returning to Pisa he published, in 1202, his great work, the *Liber Abaci*, . . . the first great mathematician to advocate the adoption of the 'Arabic notation' ". And later we read: "In 1299, nearly 100 years after the publication of Leonardo's *Liber Abaci*, the Florentine merchants were forbidden the use of the Arabic numeral(s) in book-keeping, and ordered to employ the Roman numerals or to write the numeral adjectives out in full." The interesting parallel, but in the opposite direction of legal enforcement of innovation, is that in 1801 and again in 1837 the French introduced legal penalties against those recalcitrants who still held out against the metric system.

Arabic Numerals

The system of Arabic numerals (really, as we have just seen, Hindu-Arabic) with its "place" convention — and this, not the choice of ten at all, is the real novelty and the real advantage — was thus introduced into Europe by one man, and had to fight its way for acceptance long years after he was dead. Thus a gravestone in Baden in 1371 and another in Ulm in 1388 are the first to show Arabic and not Roman numerals. Coins are more indicative: Swiss of 1424, Austrian 1484, French 1485, German 1489, Scots 1539, English 1551. The earliest calendar with Arabic figures is of date 1518. So our authority sets down; but he may be out in slight respects.

Napier and the Decimal Point

It would be tedious for the present purpose, however interesting for leisurely investigation, to pursue this. Enough to say that the first to invent the "decimal point", written by him as a comma, was John Napier of Merchiston, in his *Rabdologia* of I6I7, the year of his death and three years after the publication of his logarithms. Then in that era following the Renaissance, mathematics and arithmetic began to make the cumulative and ever-accelerating progress which we know; and so we come, by some drastic telescoping, to where this essay began, at the years 1790, 1795, 1799, the introduction of the metric system and the decimal system of currency, which Britain, having delayed so long with instinctive, characteristic and well-founded hesitation, is now considering. I propose to vindicate in the ensuing paragraphs the soundness of that instinct, to show that Britain need adopt nothing whatever from France, America or the apparently progressive though in fact mathematically reactionary change of system in South Africa, and to try in some measure to forecast the future of computation.

The Duodecimal System

The episode of Leonardo Pisano is significant. The supersession of Roman numerals by Arabic digits, and eventually, but not all at once, by the "place" and "pointshifting" system, was in its initial stage the work of one man of perception but above all of conviction and energy. This strength of conviction, but now in a new and even more progressive direction, namely that the system of Leonardo is not the final word but that the duodecimal system with appropriate notation is appreciably superior again, is held at the present time by a relatively small number of persons in the whole world. (It is true, of course, that the vast majority of the rest are entirely ignorant of the whole issue.) One may mention the Duodecimal Society of America, counting in its membership distinguished actuaries and other prominent men—and it is symptomatic that such a society should take its origin in a country devoted since 1786 (a date in which America had no mathematical standing whatever) to decimal currency, though not, and this is again symptomatic, to decimal metric; there is a Duodecimal Society of Great Britain, recently founded, small in membership and resources; while in France, home of the decimal-metric system, there is M. Jean Essig, Inspecteur-Général des Finances, whose notable treatise on duodecimal arithmetic and measures, *Douze: notre dix futur* (Dunod I 955), is taken seriously, as the foreword shows, by Membres de l'Institut in France and Belgium. This small band of convinced men increases its numbers all the time and gains successes here and there, as when, for example, the most recent and progressive American school-texts on arithmetic and algebra, at the secondary stage, devote an extensive chapter to the description and appraisement of "scales of notation", leaving the pupil in no doubt regarding the relative inferiority of the decimal system.

Yet anyone who enters into public discussion on duodecimal calculation comes at once upon the strangest circumstance. Incredible numbers of persons have been so imperfectly educated as to suppose that the decimal system is the only one that admits "place" notation and the property of shifting the "point" under multiplication or division by the base. This defect of education, amounting in the case of certain newspaper correspondents to arithmetical illiteracy, has to be combated. The fact is that any integer whatever, suitable or unsuitable, can be taken as base of the corresponding system. A younger generation of persons selected by ability knows this already, namely all those who are preparing themselves for modern electronic computation, destined as it is, in the form of new machines not yet in production but easily imaginable, to transform in a hardly recognisable way whole domains of financial and official calculation, to say nothing of the arithmetical apparatus of technology generally. For while1900-1925 was the period of the hand-operated mechanical calculating machine, and 1925 and onward that of the electrical one, from 1961 to the end of the millennium will be the era of electronic computers of every range, not merely of the large, and for certain purposes too large, ones that we see being installed in more and more places, but those of moderate size (and there will be smaller ones still) which are only now beginning to be in production. These will transform not merely arithmetic, but education in arithmetic; and a younger generation, familiar with binary and octonary systems as well as with decimal, will be sure to ask: What, reckoned in terms of time and efficiency, is the worth of the decimal system, and is there a better? We shall without doubt see this happen, probably in Russia and America almost simultaneously, while we, who of all nations in the world are in the special and most favourable position to make the change, may be left behind; may well in fact have made a belated change, only to have to make a further belated one. Of course, on the other hand, there may be financial, economic and indeed political considerations which may enforce the other, to my mind reactionary, decision; but that would require a separate study, which has in some part been done and is in any case outside my competence. But I will simply say: *political expediency is the ruin of science*.

Monetary and Metrical Units

Why are we in that special and most favourable position? Because we already have the duodecimal system with us in all but name, and to a certain but lesser extent even in notation. I refer not to electronic machines, which can convert from their idiomatic binary into any other prescribed scale, but to the numberless transactions of ordinary life, in banks, ticket offices, behind counters, on board buses, wherever and whenever there is buying and selling and giving of change. Consider a railway clerk giving tickets and change, often at top speed to a heavy queue. Does he ever think of decimal tables in handing back 5s. 7d. as change from a 10s. note on a ticket of 4s. 5d.? Not he; like hundreds of thousands of men behind counters he is a highly versed duodecimalist, though it would not occur to him to give so publicly useful a faculty so highsounding a name. I know this from having spoken recently with dozens of such men. Here is a typical comment, from a Scots bus conductor: “We get on weel eneuch; yon would muck it all up again”. Some may think they might get on weel eneuch with decimal coinage; the most manage perfectly well. There is no cogent evidence that the public wish this change in the least; though the will of the public, strong as it might be either way, is neither the only nor the chief consideration. The French, at the very height, in 1790, of their enthusiasm for liberty, equality and fraternity, so qualified equality as to set up an academic commission of the most distinguished mathematicians in the land.

However, I propose — and it is not at all original with me — a certain change, a slight one, by which in a phased gradualness, an interregnum of years of quiet habituation and consolidation, we may bring in the more efficient system. It is: to have a pound, call it R for this discussion (a stag of twelve points is a royal!), of twelve shillings, a gross of pence. It banishes at a stroke all oddments from twelve shillings and a halfpenny to nineteen shillings and elevenpence halfpenny; it is a paper note, a “royal”, that mediates between and supersedes the pound and ten-shilling note, requires no new minted coinage whatever, and is very close to one and two-thirds dollars. Call it then R1:0:0. Its half is R0:6:0 shillings. Its quarter R0:3:0. Its eighth R0:1:6. All very much as at present. The half-crown might stay for a while, but eventually might be superseded by a three-shilling piece, a “quarter”, easier than the half-crown to distinguish from the florin. Pennies and the rest are exactly the same as now. For example, except that we have this R of new value, we shall write R3:0:0; and the like as before. So also for feet and inches. There might be — I do not know whether it is suitable or not, and would not presume to dictate to the practical measurer — a new “rod” simply of twelve feet, and this would make parallelism complete. Duodecimalists should not dictate too much what is desirable; they may well leave it to practical craftsmen to find what is the best accommodation, provided only that the final outcome is indeed cast in a duodecimal hierarchy of units. Here I differ from many duodecimalists; for I believe that, if the principle is once accepted, practical and intelligent men can be trusted to find possibly an even better solution than any duodecimalist or duodecimalist society might have proposed.

General Arithmetic

However, to go further, let us pass from the monetary or metrical units and super- or sub-units to the general arithmetic of the matter. Thus, let the fraction a half itself, in whatever context, be denoted by 0:6, a third by 0:4, a quarter 0:3, a sixth on:, twelfth 0:1, where the colon (most duodecimal publications use a semi-colon) serves for the duodecimal point, and will move right or left under multiplication or division by twelve. For example, movement to the left. What is a twenty-fourth? A twelfth of a half, hence 0:06; a thirty-sixth is 0:04. A thirty-sixth of the new royal is indeed fourpence; and so on. Contrast this with the inexact and inadequate third as 0. 33333 . . ., sixth as 0.6666 . . ., twelfth as 0.083333 . . ., and so on to more turgid examples. Someone may say: What about a fifth or a tenth? Admittedly, since five does not go exactly into twelve, we shall here obtain a non-terminating duodecimal. For example, a tenth comes out as 0:12497 . . .. the last four digits forming the recurring period; but a close approximation to this is 0:125, committing the slight error, in excess, of 1/8640. (For comparison the approximation 0. 333 for one-third commits, in defect, an error of one three-thousandth.)

However, to go slightly further still. A shilling, 1:0s., is a dozen pence. Shift the colon to the right and in fact, since it is not then necessary, remove it, and write the dozen itself as *10, the prefixed asterisk (functioning like the American dollar sign) indicating that we are in a special system, that of the dozens, the meaning of the symbols being: one dozen, no units. Similarly thirteen, being one dozen, one unit, is *11; fourteen is *12, twenty-five is *21, and so on. The gross likewise is *100, meaning one gross, no dozens, no units; I will attend to names later. But all of this is just another way of writing 1:0:0 in the new R way, the kind of thing that faces us every day on a bill. Duodecimalism is nothing but this, though of course we have to know our tables, e.g. that 7 times 9 (asterisk with single-digit numbers not required) is *53, five dozens and three. But this is the smallest part, in a slightly different notation, *of the first entries in any ready reckoner*, and we have seen that already great sections of the population know these elementary tables, from habit, from serving customers and giving change. Consider the number, in decimal notation, 457. It is three gross, two dozen and one, *321. If these happened to be pence, then, in pounds, R3:2:1; in shillings, *32:1s., three dozen and two shillings and a penny. But this is to labour the habitual; we are doing this kind of thing all the time. Everyone who knows (some do not) that twelve articles at sevenpence each is seven shillings is simply saying that a dozen times seven is seven times a dozen, namely *10X7=*70 in pence, or in shillings *10X0:7s. = 7s. I showed some of this, doing some simple addition of fractions by it, to a bank teller and likewise to a stationer. The reaction was identical; each man involuntarily shielded his eye with his hand, doubtless to ward off the blinding flash of the obvious. Well, it is that some of this, in a differently couched and very uninspiring form, is taught in the chapter of school algebra dealing with "scales of notation", though often treated in such a perfunctory fashion that the pupil may be excused from regarding it, as so much tediously useless manipulation. I exclude from my condemnation those admirable American school textbooks.

We have suggested, provisionally, *10 for twelve, *11 for thirteen: for we hope eventually to use our system exclusively and to drop the asterisk. Confusion will be caused unless we devise new *single* symbols for ten and eleven; we can keep the names. Is it beyond the power of artistic typography (I suggest ↊, an inverted 2, for ten; and ↋, an inverted 3, for eleven) to invent simple, distinctive, cursive and aesthetically satisfying symbols for these two integers? The Hindus had to invent *all ten* of their symbols; while I could show many unsuspected situations in ordinary arithmetic where an alternative ten, at least, would have been valuable. On the Chinese and Japanese abacus there were and are *two* ways of expressing five, appropriate to different situations. For myself, I do my calculations with no great need for symbolic representation, but the above inversions of 2 and 3 served me well enough. Certain duodecimal societies, as well as a good many idiosyncratic individuals, have advocated various symbols, quite commonly X or χ for ten, E for eleven, and so on. This will not do: letters of the alphabet must be kept for algebra, not arithmetic; let us think of the confusion in trying to write in such a way "ten times X". So also for nomenclature.

For myself, I do not depend much on auditory impression for number, but thinking of the Scots"twal" I sometimes imagined "twel-one", "twel-two", and so on for thirteen, fourteen and the rest; but of course in dictation one would mention "asterisk" and call out, just as we do in decimal, "one one", "one two" and the like for *12, *12, etc. There should be no difficulty here. Once again, duodecimalists should not prescribe too much for others in this matter; language and linguists should be able to find, as the French language does with never-failing felicity, euphonious and idiomatic equivalents for any new entity that may arise. For example Icelandic also, when faced with the necessity of finding words for radio, television and so on, merely drew on its own resources. Let the principle be once stated; we can weigh later the merits of the different suggestions.

As for early education in the properties of numbers, it is evident that twelve is a far more interesting number than ten, and two sets of six or twelve coloured blocks, to be arranged in various ways by twos, threes, fours and so on, would show to the growing mind the mutual relations of small integers better than any of the usual devices based on ten, some of them in any case open to criticism. Above all, no dependence on fingers.

This will be enough of description for a first summary. A graduated set of simple exercises would lead anyone, even a child, easily into this realm thus simplified. But it will be asked: are the reasons for change aufficient, both qualitatively and quantitatively, to justify, so late in the history of the world, such a radical transformation of mental habit and customary practice? The replies are: First, it is very early in the history of the world. Second, that in our case at least, the change is not radical; we do much of it already every day. Third, partly qualitative, that since the dozen, helped by its multiples and submultiples, is so extraordinarily superior to ten in all that concerns parcelling, packaging, arrangement, subdivision, to say nothing of a host of applications which could be cited from mathematics, the practical use of the dozen and its adjuncts should go hand in hand and step for step with the corresponding numerical use; and this implies the duodecimal system and no other.

Finally, the quantitative advantage. To begin with, the multiplication tables are simpler than the decimal ones; there are only 55 (duodecimally *47) essential products to be learned, exactly the same number as have to be learned in our school tables up to twelve times twelve—and observe that even there we had to go to the dozen. (Incidentally in duodecimal the square of *11 is *121, of *12 is *144, with different numerical meaning, of course.) For multiples of 2, 3, 4, 6, 8, 9 and 10 we see in the last digits a simple and useful periodicity. For example, the four times table: last digits 0, 4, 8, 0, 4, 8, 0, 4, 8, and so on; the three times table: last digits 0, 3, 6, 9, 0, 3, 6, 9 and so on. Tests for divisibility: for divisibility by 2, 3, 4, 6, look at the last digit only; by 9, 16, 18, the last two; and so on.

Duodecimal fractions, as we indicated by a few examples earlier, are in the usual fundamental ones of low denominator *remarkably simpler* than decimal. Consider the table below:

Fraction	*Decimal*	*Duodecimal*
1/2	0.5	0:6
1/3	0.3333	0:4
1/4	0.25	0:3
1/5	0.2	0.2497
1/6	0.1666	0:2

1/8	0.125	0:16
1/12	0.0833	0:1
1/24	0.04166	0:06

Tables of successive halvings, as for example the table for conversion of sixty-fourths into decimals that hangs on the wall of many tool shops, shows comparisons such as the following five: thus

Fraction	*Decimal*	*Duodecimal*
25/64	0.390625	0.483
27/64	0.421875	0:509
29/64	0.453125	0:553
31/64	0.484375	0:599
33/64	0.515625	0:623

With only three digits, the duodecimal fractions are all exact. Comment is needless.

But the final quantitative advantage, in my own experience, is this: in varied and extensive calculations of an ordinary and not unduly complicated kind, carried out over many years, I come to the conclusion that the efficiency of the decimal system might be rated at about 65 or less, if we assign 100 to the duodecimal.

Others (but so far I have not heard of even one such investigator) might arrive at a slightly different estimate; but I am certain that in every case a marked superiority for the duodecimal system would be established. If such a waste of time and effort (about 350 hours lost in every 1000) were found to be trickling away in any department of a modern production unit, a time-and-work study would at once be set up. Some altruist might even come in with a take-over bid. Is it to be doubted that such time, saved and turned to more productive ends, social or economic, would give an advantage much outweighing any advantage assumed to accrue now, at this late stage of decision, from moving over to the decimal system; an assumption moreover implying, since the decision has taken about 150 years to make, that the new status of things would last for at least another 150 years.

Nothing stands still, not even arithmetic. That arbitrary division of time, the second millennium, is approaching, heralded as it has been somewhat prematurely from a distance of forty years; and no doubt a few thousands of superstitious decimalists will sit up on that eve to await the new dawning of heaven and earth. In the interim there is bound to be incredible technological progress, enough possibly to give us some glimpse of "the uses of leisure". Among these novelties the transition from a defective system of numeration and metric, to a new one, attainable by easy and gradual phase, will be viewed in remote retrospect as one of the most ordinary pieces of belated tidying-up that ever was delayed for so long past its due time. It will be viewed, indeed, by the future historians of mathematics, as completing the work of Leonardo, in a direction which, with the added knowledge of 800 years, he would have approved.

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